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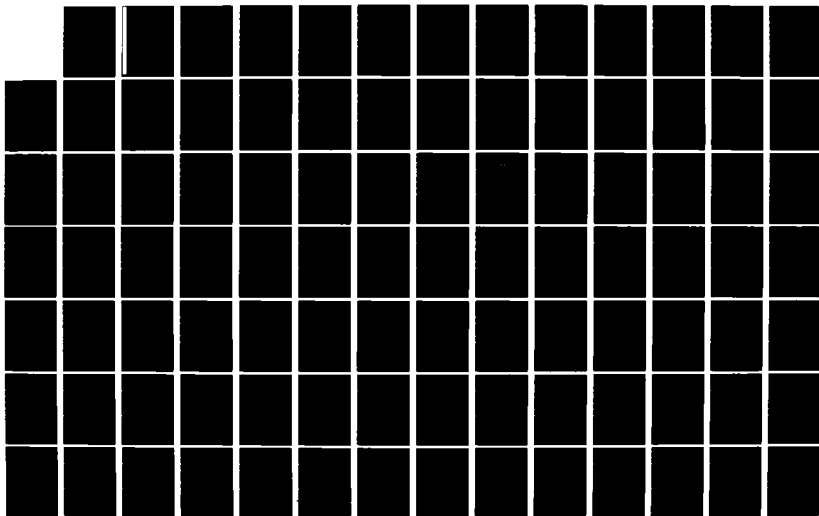
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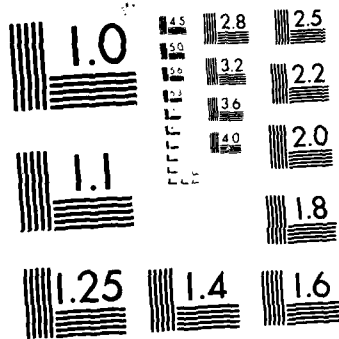
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APPLICATION OF THE WAVY MECHANICAL FACE SEAL  
TO SUBMARINE SEAL DESIGN

BY

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REPORT NO. ME-117(82)ONR-414-1

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APPLICATION OF THE WAVY MECHANICAL FACE SEAL  
TO SUBMARINE SEAL DESIGN

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) In this report the results of theoretical and experimental investigations on the effects of waviness on mechanical seal performance are presented. In addition, the design techniques, design, and test results for a nine-wave small scale submarine seal are presented.  (continued on reverse)		

The nine-wave seal has been shown to operate satisfactorily in preliminary testing. Problems with fabrication, stiffness, bonding, and oil system sealing were encountered and solved. Test results show friction to be somewhat higher than predicted. Leakage is very low. Further tests are being conducted. Further testing on balanced parallel face seals is reported. Details of test machine modifications required to operate in a simulated submarine environment are given.

Complete details of the design methodology and the details of the nine-wave seal design are given. The design of a device to create variable misalignment during testing is also given.

The moving waviness concept has been applied to the design of a gas seal. Certain advantages of using moving waviness compared to other designs are noted. A complete theoretical-numerical analysis of moving wave gas seal performance is made. Parameter studies are performed. Results show very low leakage and friction. Potential problems are discussed.

Progress was made on solving the general problem of determining the precise contact pattern of two seal rings pressed together. This problem is important in assessing seal leakage problems. A new, general, coupled (in- and out-of-plane), ring finite element was developed and checked. Simple beam contact problems were solved using finite element methods. The final solution requires combining the finite element and the contact problem.

Conclusions and recommendations are made.

APPLICATION OF RESEARCH TO THE NEEDS  
OF THE U.S. NAVY

Mechanical face seals are used in numerous applications in Naval machinery. These applications range from propeller shaft seals to boiler feed pump seals. In such equipment the mechanical seal plays a vital role. When such seals fail, repair is costly both in terms of lost time and direct costs, so any improvement in seal life and reliability would be of significant benefit.

As more advanced equipment is designed, it is sometimes difficult to achieve desired performance in more severe service environments with the present state of the art of seal design. Thus, an improvement in seal technology would serve this important application.

One objective of the research herein is to further the understanding of mechanical face seal lubrication phenomena. Another objective is to develop the capability of designing contacting face seals having a longer life, greater reliability, and for extreme environments. The immediate objective herein is to apply the knowledge gained to the design of a small scale submarine type seal. Thus, the objectives of this research are compatible with mechanical face seal needs for Naval machinery.



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# LIST OF SYMBOLS

$a$	Ring cross sectional area - $m^2$
$A = \frac{EJ_x}{GJ_\theta}$	Stiffness ratio - dimensionless or area - $m^2$
$b$	Fraction of seal subject to fluid pressure
$B = \frac{r_o^2 - r_b^2}{r_o^2 - r_i^2}$	Balance ratio for an outside pressurized seal
$c$	One-half maximum roughness height - $m$
$d$	Diameter of gas pressure hole and parameter as defined - $m$
$e$	Distance between gas pressure hole center and centroid - $m$
$e_1 e_2$	Eccentricities as defined - $m$
$E$	Youngs modulus - $N/m^2$
$E( )$	Expectancy operator
$f(h_s)$	Roughness distribution function
$F$	Force - $N$
$G$	Shear modulus - $N/m^2$
$h$	Nominal film thickness - $m$
$h_n$	Amplitude of the $n$ th harmonic - $m$
$\bar{h} = \frac{h}{c}$	Dimensionless film thickness
$h_o$	Nominal film thickness or minimum film thickness (between mean roughness heights) - $m$
$H$	Film thickness in difference equation - $m$

$I_x, I_y, I_{xy}$	Approximate values of $J_x J_y J_{xy}$ based on straight beam theory
$J_x = \int_A \frac{y^2}{1 - x/r_c} dA$	Stiffness constant about x axis for ring cross section - $m^4$
$J_y = \int_A \frac{x^2}{1 - x/r_c} dA$	Stiffness constant about the y axis for ring cross section - $m^4$
$J_{xy} = \int_A \frac{xy}{1 - x/r_c} dA$	Stiffness constant - $m^2$
$J_\theta$	Torsional stiffness constant for ring cross section - $m^4$
$K$	Ratio of contact radius to inside radius or stiffness matrix
$\ell$	Length of beam element - $m$
$m_\theta, m_{\theta_0}$	Distributed moment and amplitude on seal ring - $N \cdot m/m$
$m_r, m_\theta, m_v$	Mass flow rates $kg/s$
$M_x, M_y, M_\theta$	Ring moments - $N \cdot m$
$n$	Number of the harmonic or number of waves around seal face or normal direction
$N_\theta$	Ring normal force - $N$
$p, P$	Fluid pressure - $N/m^2$
$p_c$	Cavity pressure - $N/m^2$
$p_g$	Gas pressure amplitude causing waviness - $N/m^2$
$p_i, P_i$	Seal inside pressure - $N/m^2$
$p_m, P_m$	Pressure at asperity contact--equals compressive strength - $N/m^2$
$p_{ma}$	Average mechanical contact pressure - $N/m^2$
$p_o, P_o$	Seal outside pressure - $N/m^2$

$P_s, P_s$	Shear strength of asperities - $N/m^2$
$P_{sp}, P_{sp}$	Spring pressure on face - $N/m^2$
$Q$	Total leakage for the seal - $m^3/s$
$Q = p^2$	Pressure squared used in A.D.I. method
$\bar{Q}$	Assumed known value at previous time step
$Q_{in}$	Fluid leakage across the inside face
$Q_{out}$	Fluid leakage across the outside face
$r$	Radial coordinate
$r, \theta, z$	Seal coordinates
$r_b, R_b$	Seal balance radius - m
$r_c$	Radius to centroid of seal ring - m
$r_f$	Friction radius - m
$r_i, R_i$	Inside radius of seal - m
$r_o, R_o$	Outside radius of seal - m
$R$	Radius to centroid of seal ring - m
$R$	Radius of the control volume center - m
$S_u, S_y, S'_e$	Ultimate strength, yield strength, fatigue strength - $N/m^2$
$S$	Stiffness of the gas seal - $N/m$
SCFM	Standard cubic feet per minute
SCMM	Standard cubic meter per minute
$t$	time - s, bond thickness - m
$T_q$	Seal friction torque - $N \cdot m$
$u, v, w, \phi$	Ring displacements
$U$	Velocity - m/s or energy - $N \cdot m$

$v, v_0$	Ring centroid displacement and amplitude - m
$V_x, V_y$	Ring shear forces - N
$V_{r,\theta,z}$	Velocity components
$x, y, \theta$	Ring coordinates
$w(r)$	Seal wear as a function of radius
$W$	Total load support - N
$W^*$	Required load support - N
$\alpha_1\alpha_2$	Angle as defined
$\beta_1\beta_2$	Angle as defined
$\Delta a$	Differential area of control volume
$\Delta A$	Differential area in difference form
$\Delta \theta$	Differential change in angle of control volume
$\Delta R$	Differential change in radius in finite difference form
$\Delta T$	Time step in A.D.I. method
$\nabla$	Gradient
$\gamma$	Angle as defined
$\delta$	Displacements
$\eta$	Viscosity - N • s/m <sup>2</sup>
$\theta$	Angular coordinate or flow variable
$\theta$	Angular coordinate
$\Lambda$	$6 \omega \eta$
$\mu$	Friction coefficient or Poisson's Ratio
$\xi, \xi'$	Angles as defined
$\rho$	Density - kg/m <sup>3</sup>



$\tau_a$	Asperity shear stress - $N/m^2$
$\tau_f$	Fluid shear stress - $N/m^2$
$\phi$	Rotation of seal ring about its centroid
$\phi_0$	The maximum rotation of the seal
$\omega$	Angular speed
$\Omega$	Angular speed of waviness - $1/s$
%	Percent of load supported by fluid pressure
-	All symbols with bar are dimensionless as defined.

## CHAPTER 1

### INTRODUCTION

#### Mechanical Face Seals

Applications of face seals range from water pumps to compressors, to power pumps, to propeller shafts. In many applications the reliability of the mechanical seal is of the greatest importance to the reliability of the equipment itself.

Mechanical face seal technology has been steadily improving over the past several decades. However there still remain demands for seal performance which have not been met. One such application of note is the submarine propeller shaft.

Such demands on a particular technology can often be satisfied by first improving the technology. In the case of mechanical face seals, the main barrier to advancement has been that the mechanics of seal operation are not well enough understood to be able to reasonably anticipate seal performance as a function design parameters.

During the past five years of this research program, much has been learned about controlling the hydrostatic and hydrodynamic mechanisms which enhance face seal operation. In this work, this knowledge is applied to the design of an improved small scale submarine shaft seal. Theory, design, and test results are presented. The results show promise that significant improvements in submarine shaft seal design are possible, and the application of such designs could greatly increase the length of trouble-free shaft seal service.

#### Seal Lubrication

As background, the mechanical face seal consists basically of two annular rings which rotate relative to each other and which are pressed together by spring and fluid pressures (see Figure 1-1). In conventional seals, the surfaces that rub

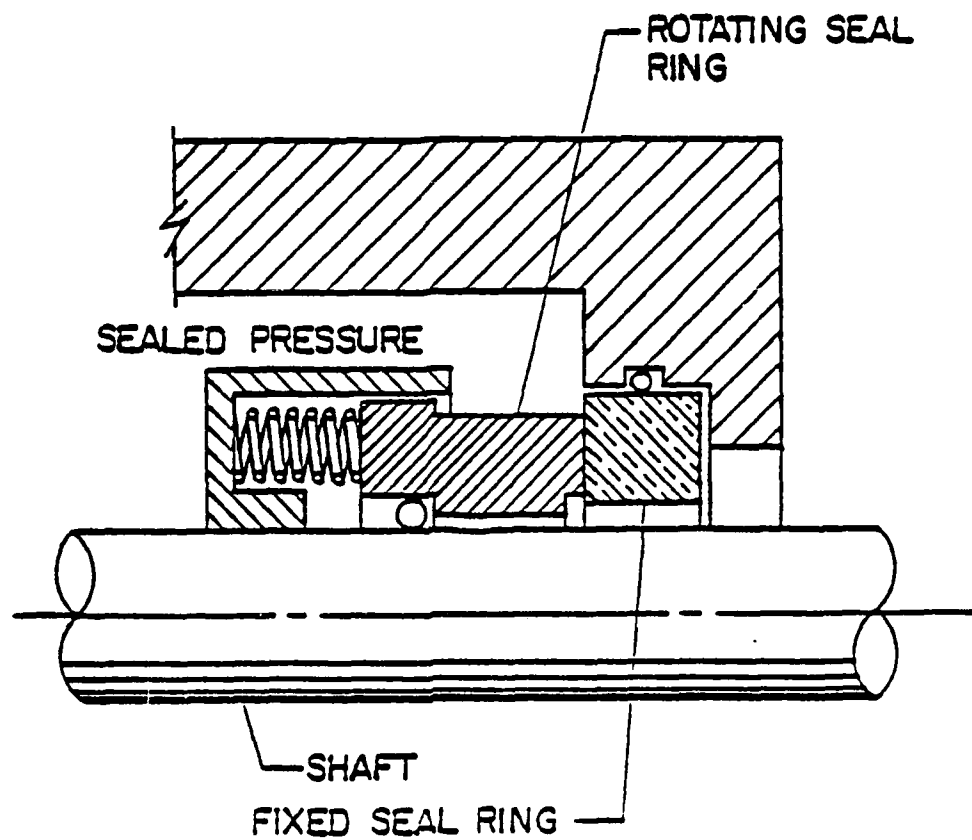


Figure 1-1. Mechanical Face Seal.

together are generally manufactured as flat as possible initially so as to minimize leakage. The effective gap between the faces is ideally quite small (order of  $1\text{ }\mu\text{m}$ ) so that leakage flow across the faces will be quite small. The difficulty in designing a mechanical seal is in maintaining the gap at a very low value while at the same time providing a definite lubricant film between the faces.

The load that must be supported at the faces of a mechanical seal is due primarily to loading caused by the sealed pressure. The load support at the faces is derived from fluid pressure and mechanical pressure. If the fluid pressure at the faces is large enough to support all of the load, then there will be no contact and no adhesive wear.\* If none of the load is supported by fluid pressure, the load must be carried by mechanical contact, and the wear rate will be large.

In practice, conventional seals often operate at one of two extremes. At one extreme, a large gap will be created by hydrostatic or hydrodynamic pressure or distortion, all of the load will be supported by fluid pressure, and the seal will leak a lot and wear very little. At the opposite extreme, the gap will close completely. Leakage will be low but only a fraction of the load will be carried by fluid pressure, and wear and heat generation will increase.

Based on the above, it can be concluded that an effective seal should operate between these two extremes--having both adequate fluid pressure load support and low leakage. The seal should operate so that it just touches to minimize leakage but such that the load is carried by fluid pressure. To do this

---

\*There may still be abrasive or corrosive wear even if the surfaces do not touch.

requires that any fluid pressure generation mechanism used to provide load support to the seal must be very carefully controlled. At present in commercial seals, this is left primarily to chance and sometimes seals operate at one of the undesirable extremes mentioned.

In this research program, attention has been devoted toward studying the effects of waviness as a source of controlled hydrodynamic and hydrostatic load support. Waviness was selected because it is controllable. In this present work, waviness has been applied as the basis for the design of an improved small scale submarine shaft seal.

#### Background

ONR-sponsored research on mechanical seals had been conducted for five years prior to the beginning of the submarine seal design phase described herein. The work being reported evolved from various discoveries over this five year period, and the past work must be reviewed to better understand the nature of the current work.

As a starting point for this Navy research program, the effects of waviness on seal performance were modeled in some detail. In the first annual report for this project, Reference [1], this general problem was solved using a one-dimensional theory. In the second annual report, [2], the much more complex two-dimensional solution to the above problem was solved. The effects of waviness, roughness, asperity contact, wear, cavitation, and elastic deflection were included in this model. Using this model, predictions were made for the relative wear rate, friction, and leakage as a function of roughness, waviness, speed, size, pressure, viscosity, and material.

A number of conclusions were reached based on these first two annual reports:

- 1) The effects of roughness on hydrodynamic lubrication are not completely understood. Certain fundamental questions remain concerning the roughness model used.
- 2) As to the potential of utilizing hydrodynamic effects caused by parallel face waviness to advantage by design, the results show that wear rate and friction can be greatly reduced while maintaining leakage at acceptable levels.
- 3) While a comparison of predicted results to experimental results given in the literature is generally good, data contained in the literature is incomplete, so more complete experimental data are needed for comparison.
- 4) In low viscosity or heavily loaded applications where some touching is expected to occur, waviness will wear away with time and any benefit derived will be lost unless something is done to counteract this effect.
- 5) Based upon data for some commercial seals and using the model, it was determined that there was insufficient accidentally caused waviness to produce significant hydrodynamic effects in water. One cannot generalize to say that such effects do not occur in commercial seals. However, using the model the question can be answered on a case by case basis.

Item 1) was treated extensively in the third annual report [3]. Even after this analysis certain fundamental questions remain concerning how to deal with roughness in lubrication problems. However, this thorough analysis led to conclusions allowing certain simplifying assumptions discussed in the fourth annual report [4].

Item 2) was also treated extensively in the third annual report [3]. A methodology for the design of a wavy face seal was developed and applied. Theoretical results showed large reduction in friction and wear rate compared to conventional designs whereas leakage could be controlled.

Concerning Item 3), the second and third annual reports [2,3] describe a test apparatus designed to test the wavy seal theory. This apparatus has been in operation for more than three years and many tests have been conducted. These test results are reported in the fourth and fifth annual reports [4,5].

Early in the test program it was observed that the type of waviness which can be practically applied is not of the radially parallel type. Waviness generally consists of alternating tilt plus radially parallel waviness. Based on these considerations, a new model for predicting performance was developed and appears in the fourth annual report [4].

Concerning Item 4) above, a solution to this problem was first proposed in the first annual report [1]. It was proposed to move the waviness slowly around the seal so that whatever wear occurred would be uniformly distributed. Then the shape of the wave would be preserved and tests using a constant wave could be made. The concept is illustrated in Figure 1-2. This concept was incorporated into the test apparatus and is described in detail in References [2] and [3]. In the fifth annual report [5], a new concept to move the waviness with no internal moving parts is described in detail.

In the fifth annual report [5], additional experimental results using waviness are presented, the wavy seal model is further improved, and theory and experiment are compared. The results using a new concept, that of a self-generating seal profile, are reported. Results from tests of the effects of radial taper and high temperature environment are also reported and compared to theory.

In summary, both the theoretical and experimental basis for applying waviness to a mechanical seal to reduce friction and wear were well established during these first five years. In the present work, the waviness concept is applied to the design of a long life submarine shaft seal as described later.

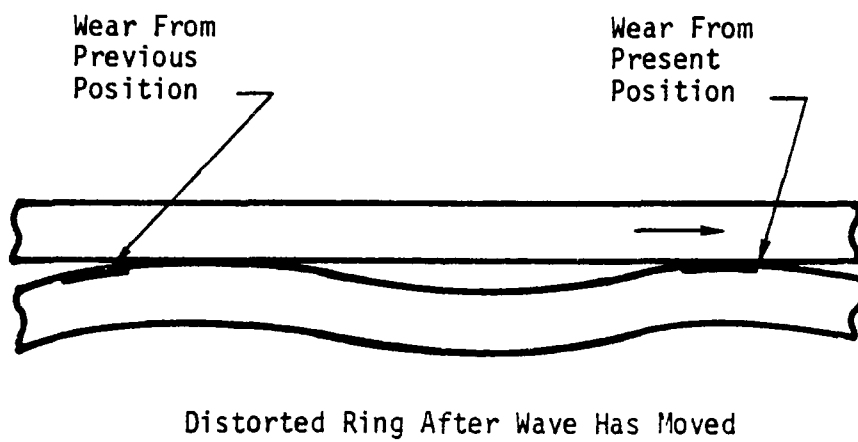
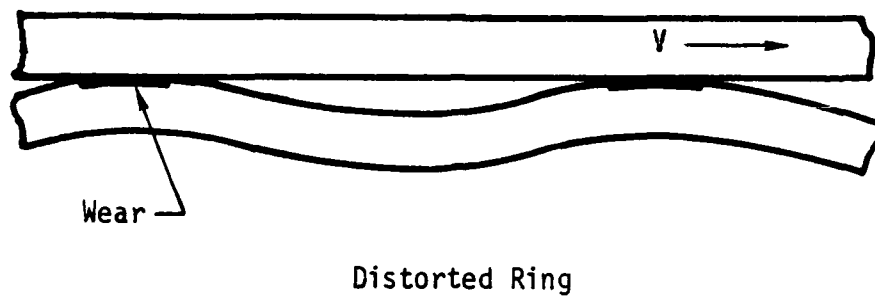


Figure 1-2. Moving Waviness Concept.



The results developed to date and discussed in the five annual reports are also presented in several papers and theses [6-24].

### Wavy Face Seal

The concept of waviness is that the film thickness varies in some fashion circumferentially around the seal. Generally speaking, film thickness may vary radially as well as tangentially.

$$h = h(r, \theta) . \quad (1-1)$$

In the present work interest is focused upon film thickness shapes of the following functional form

$$h = h_0 + f(r) \cos n\theta . \quad (1-2)$$

At any particular radius  $r$  the film shape is periodic with  $n$  waves around the seal and is therefore wavy. However, film shape can also vary in some general manner with  $r$ .

If  $f(r) = \text{const}$  then the faces are always radially parallel. This component of film thickness variation is commonly termed waviness. If  $f(r) \neq \text{const}$ , then the faces are not in general radially parallel.  $f(r)$  is referred to as tilt. Thus, the film thickness shapes of interest are combinations of waviness and tilt. Since at any radius the film thickness is wavy, the combination of waviness and tilt defined above will also be called a wavy film shape.

The reason for choosing film shapes as described by Equation (1-2) as a subject for study is that these shapes can conveniently be generated by planned mechanical distortions in a seal ring, and the shapes also include common modes of unplanned distortion found in operating seals. For example, generally seals undergo a uniform tilt due to pressure and thermal deformation. When rings are loaded by any nonaxisymmetrical load they become wavy as described by Equation (1-2).

Now, for the sake of illustration, assume a seal has a wavy film thickness shape given by Equation (1-2) where  $f(r)$  is a linear function of  $r$ . This gives a film shape as shown in Figure 1-3 where for each of the  $n$  periods the seal touches all across at one point and is radially convergent  $\pi/n$  radians away. At any radius, the seal is wavy circumferentially. The waviness enhances hydrodynamic effects and the radial convergence enhances hydrostatic effects.

It is this shape which previous research [1-5] has shown has the greatest potential to reduce friction and wear while holding leakage at very low levels. This is the film shape used for the design of the small scale submarine seal.

#### Submarine Seal

The general objectives of the present three-year contract phase are:

- 1) to conduct further experiments using moving waviness to further the understanding of this concept.
- 2) to refine mathematical models already developed so as to be able to better predict performance.
- 3) to demonstrate in a practical way the use of the concept of moving waviness to reduce friction and wear in mechanical seals.
- 4) to undertake the development needed to be able to demonstrate the applicability of the concept to submarine shaft seals and to design a full scale long life seal.
- 5) to explore other uses of the waviness concept such as for gas seals and to create better methods by which the concept can be applied.

From these objectives, the following specific tasks have been defined.

- 1) Design, fabricate, and test a nine wave optimum seal.  
As shown in the previous annual report [5], nine waves

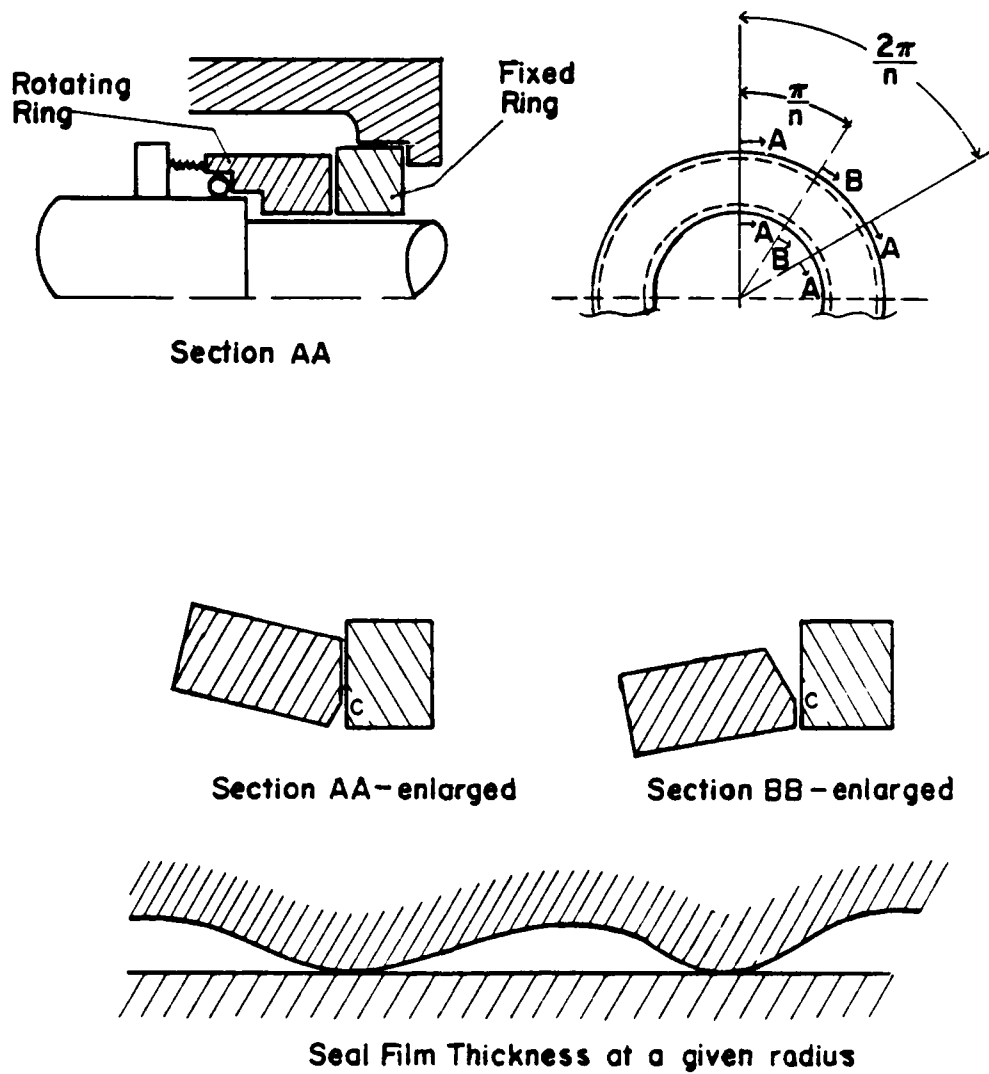


Figure 1-3. Mechanical Seal with Waviness and Tilt.

are needed to minimize changes in tilt with changes in pressure and speed.

- 2) Operate the seal for an extended period of time under simulated submarine operating conditions including seawater and changing speed, pressure, and alignment.
- 3) Evaluate compliancy of existing and proposed submarine seals.
- 4) Conduct friction and wear tests on carbon and hard face materials.
- 5) Seek alternate methods of applying moving waviness.
- 6) Perform analysis of a moving wave gas seal.

Progress has been made on all of these tasks, and the report to follow details the research efforts and achievements for the period December 1, 1980 through June 30, 1982.

## CHAPTER 2

### EXPERIMENTAL RESULTS

A nine-wave seal has been designed, fabricated, and tested during this reporting period. The test apparatus has been modified to automatically change operating conditions, and a 75 percent balance ratio test has been completed. In this chapter these results will be reported.

#### Nine-Wave Seal

Design--The first item to be reported is the design of the nine-wave seal (the design procedure is discussed in Chapter 3; the design itself is discussed here). Figure 2-1 shows the assembly drawing of the nine-wave seal. Starting at the left, one of three sinusoidally varying pressures generated by the waviness drive unit [5] is directed into two (180° apart) of six pressure channels ① in the waviness cylinder ②. The pressurized fluid then passes through the pressure coupler ③ and is ported to one of three circumferential channels on the left end of the waviness adapter ④. Eighteen smaller passages, connected to each of the three circumferential channels, terminate at pressure pockets ⑤ located on the inside and outside diameters of the waviness adapter ④ (right-hand end). The 54 pressure pockets (three sets of eighteen) apply a waviness pressure to the 54 "fingers" of the nine-wave seal ⑥.

Figure 2-2 shows a modified isometric cross section of the nine-wave seal. The pads shown are labeled with a ①, ② or ③ depending on the termination point of the three sinusoidally varying pressures, i.e., one pressure, say,  $P_1$ , exerts a wave producing load on all pads labeled with a ①. The load on all outer pads is directed outward on the inside radius and the load



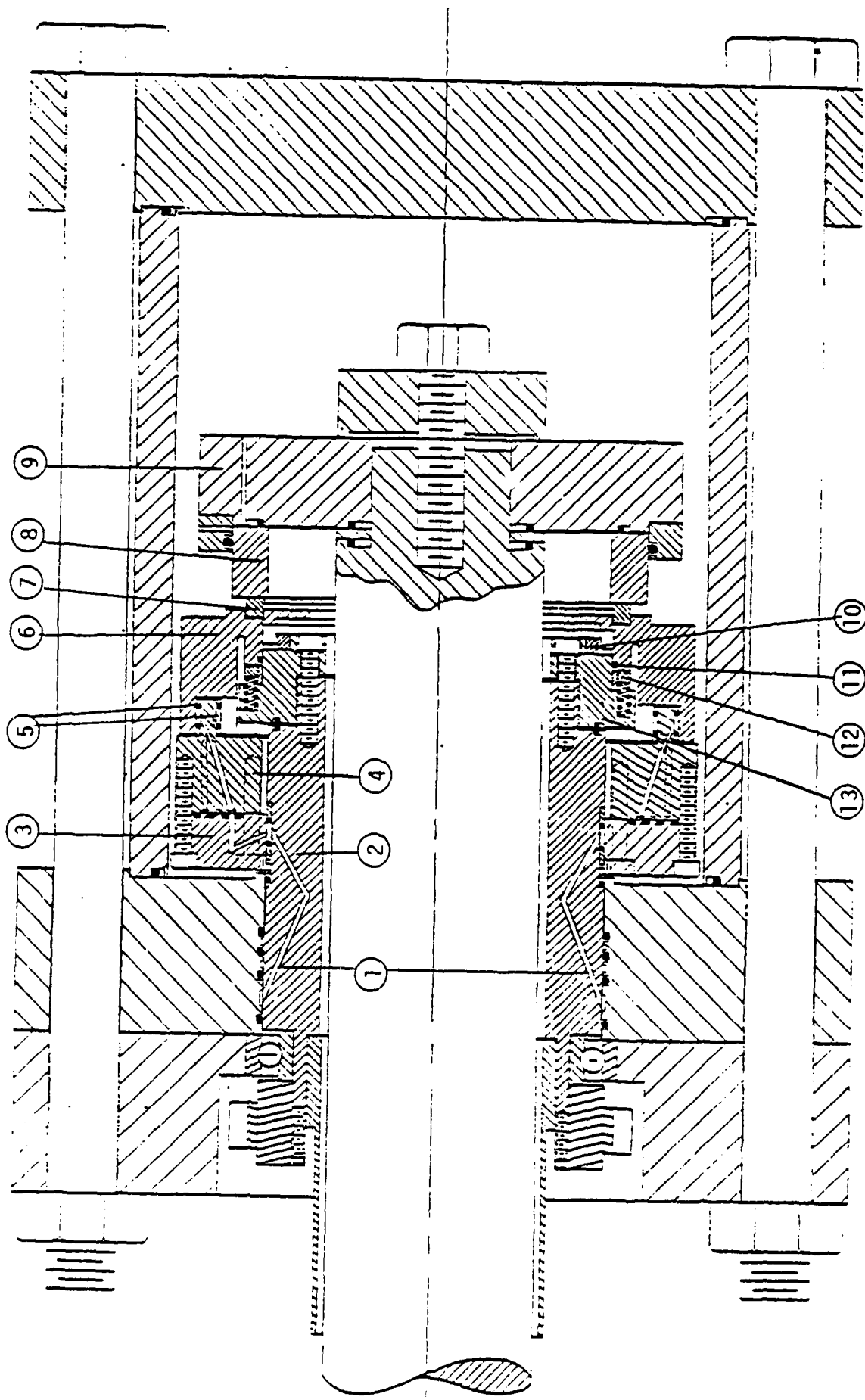


Figure 2-1. Nine-Wave Seal Assembly View.

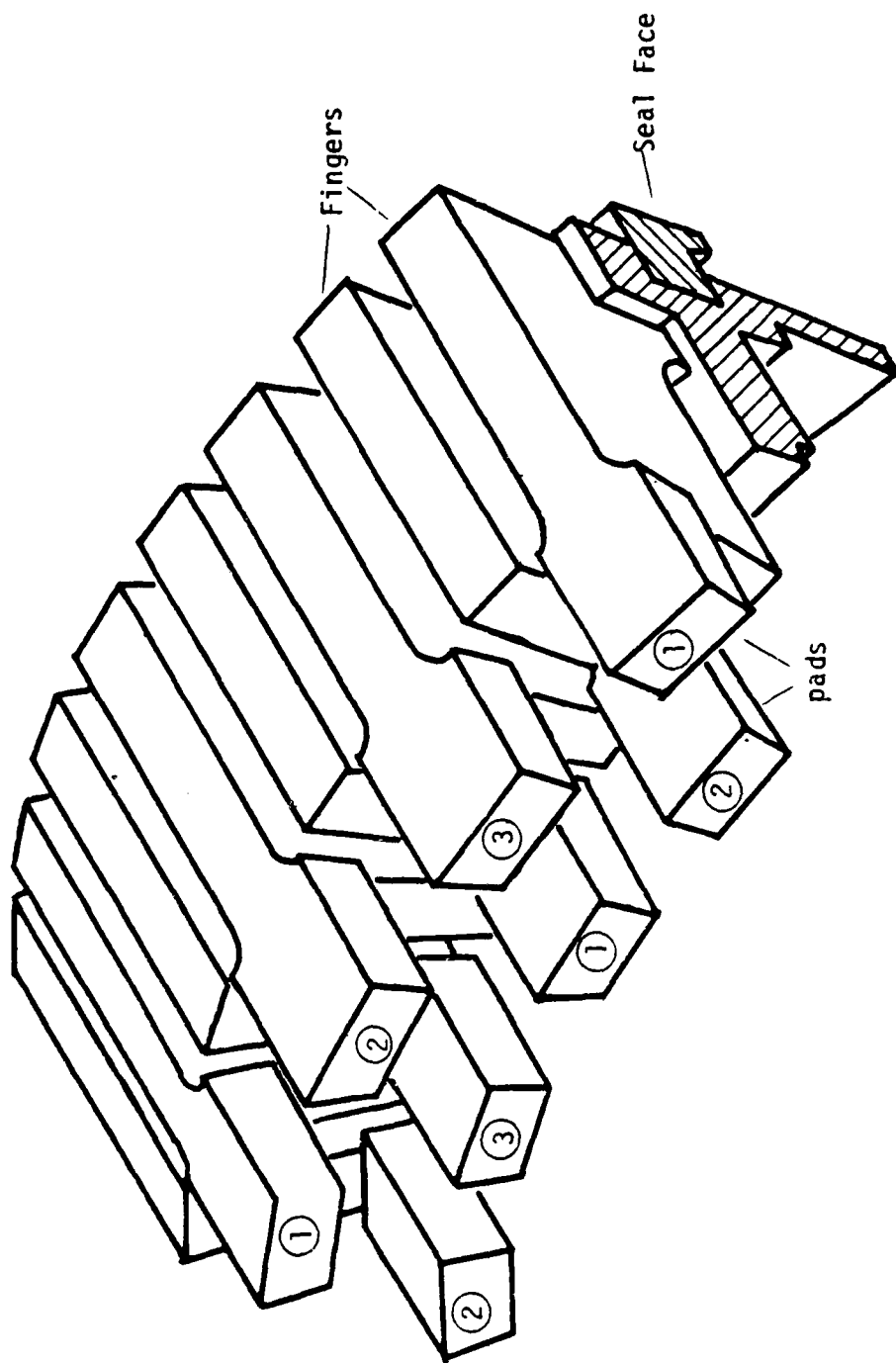


Figure 2-2. Nine-Wave Seal - Modified Isometric Cross Section.

on all inner pads is directed inward on the outside radius of the corresponding pad. With this configuration three sets of nine waves each can be imposed on the face of the carbon insert. As discussed previously [5], by sinusoidally varying the three separate pressures (and waves) with time, the result is one set of nine waves which moves circumferentially around the seal with time.

The irregular shape of the nine-wave seal (cross section) is due in part to the adjustments made on the geometry so as to obtain a zero moment design, i.e., there is no rotation of the seal about a circumferential axis due to pressure variation. Other reasons include the need for a low stiffness, so that the wave could more easily be transmitted to the carbon insert (7), and the need for the centroid to be placed at such a location so that the wave contact point, between the carbon insert (7) and the silicon carbide seat (8), is radially located for optimum seal performance.

The secondary seal (11) is located at the left end of the nine-wave seal on the inside diameter. Springs to the left of the seal, housed in the seal ring spring retainer (13), provide preload through the spring seat (12). The balance ratio for the design shown is unity.

The carbon insert (7) is a Pure Carbon P658RC material which is epoxied into the nine-wave seal using 3M 1838 B/A adhesive. It should be pointed out at this time that the carbon insert, as shown in Figures 2-1 and 2-2, has a small lip on the inside radius. This lip has a slight relief ground into it, approximately 76  $\mu\text{m}$  deep in the axial direction and 500  $\mu\text{m}$  deep in the radial direction, for purposes of wear measurements.

The drive ring (10) is designed so that the primary ring can float and align itself but cannot rotate about the shaft axis. The nine-wave seal therefore takes its alignment from the face of the rotating secondary ring. The secondary ring is a Carborundum KT<sup>®</sup> silicon carbide material which is also of zero moment design.



Modifications--Modifications to the overall design were made in light of the results obtained from preliminary testing. Figure 2-3 shows the original design cross section of the nine-wave seal. The original cross section shape, as mentioned earlier, was designed to meet specific requirements, and as a result, does not adhere closely to the assumptions used for the ring equations used to predict waviness. Bench tests to measure waviness on the carbon face showed that less than half of the needed waviness was present. Torsional deformation is predominant and for this particular shape and mode of deflection, warping becomes of considerable importance and as a result increases the stiffness. The addition of a warping factor to the design analysis did indeed show that the original cross section was much too stiff. To reduce the stiffness and maintain a zero moment design the cuts as shown in Figure 2-4 were made to the seal.

Another problem encountered was that of the epoxy bond between the carbon insert and the seal. The original design called for a 0.0254 cm (0.01 in.) bond thickness around the carbon insert. Static tests done with 100 percent sealed pressure and with a convergent taper (400  $\mu\text{m}/\text{m}$ ) lapped into the carbon face showed that the epoxy bond "creeped" in 24 hours. Final traces of the radial profile showed no taper at all. As discussed in detail later, several bonding techniques were experimented with. These included the use of Loctite Speedbonder 319, with and without a primer, Loctite Superbonder 420 with the addition of two stainless steel rings for the inner and outer diameter of the carbon insert to give a near zero clearance fit, and the use of the 3M 1838 B/A epoxy also with the same stainless steel rings. Static and dynamic tests on the seal under water pressure and independent tests on the strength of each adhesive showed that the 3M 1838 B/A epoxy with the stainless steel spacer rings was the best method of bonding.

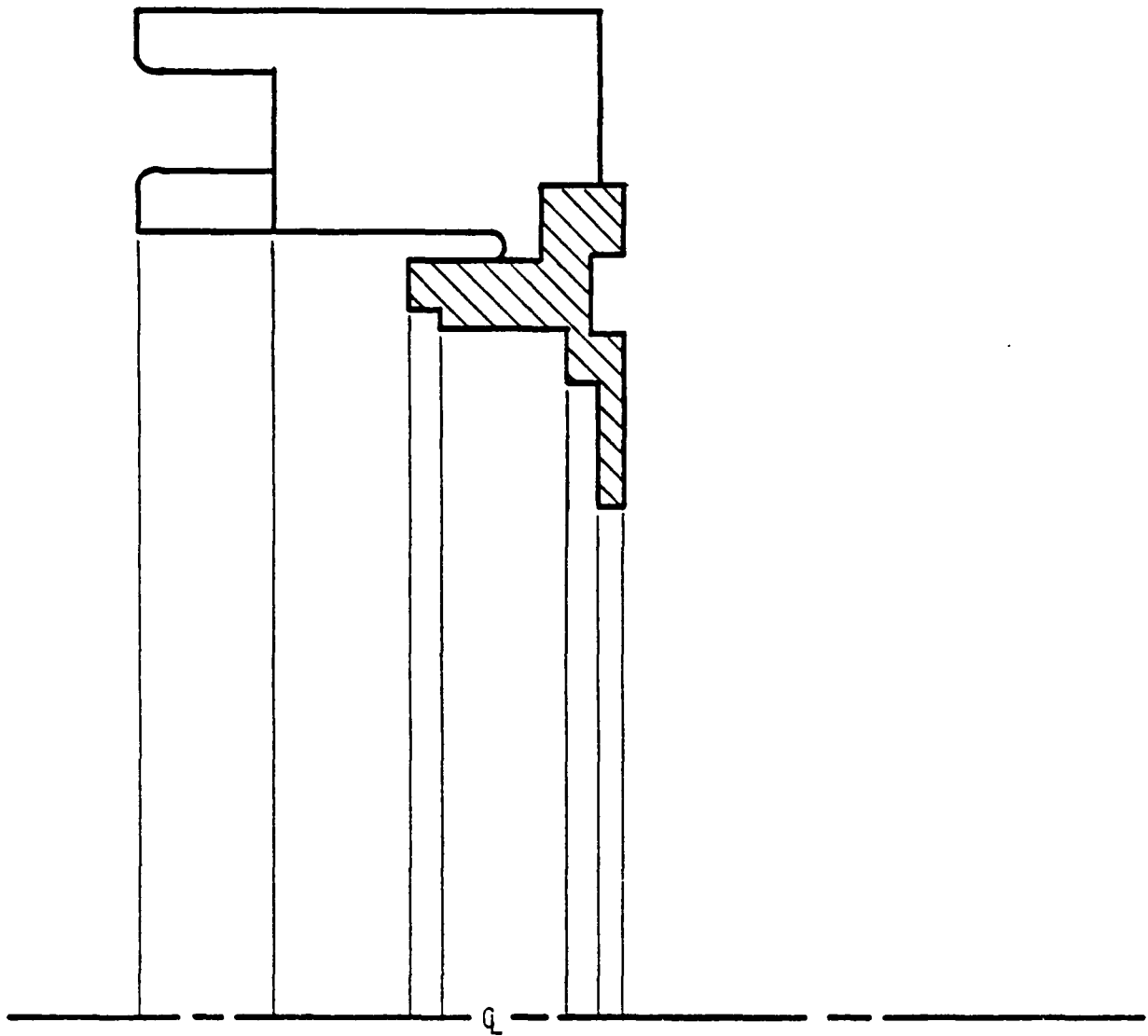


Figure 2-3. Original Design Cross Section.

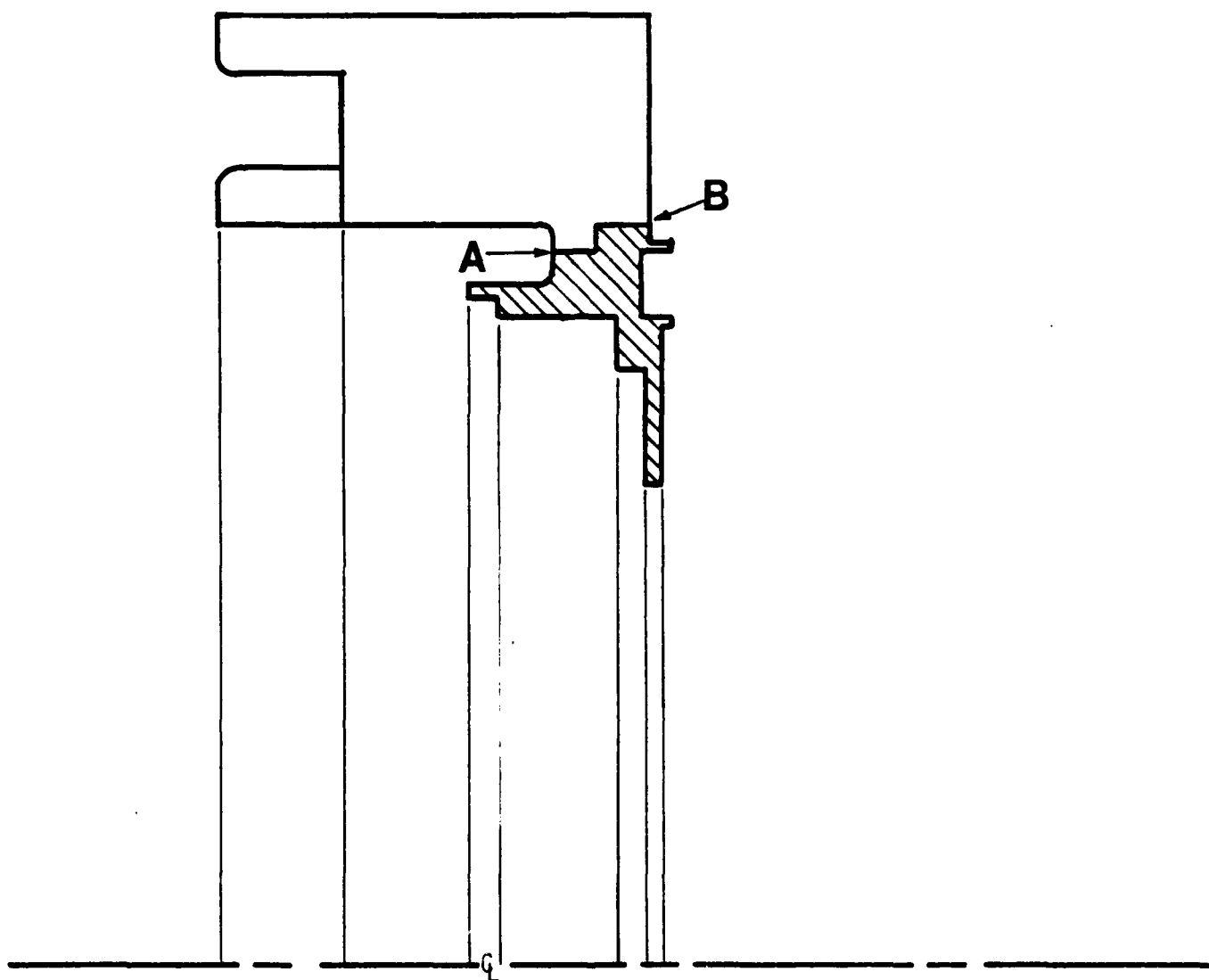
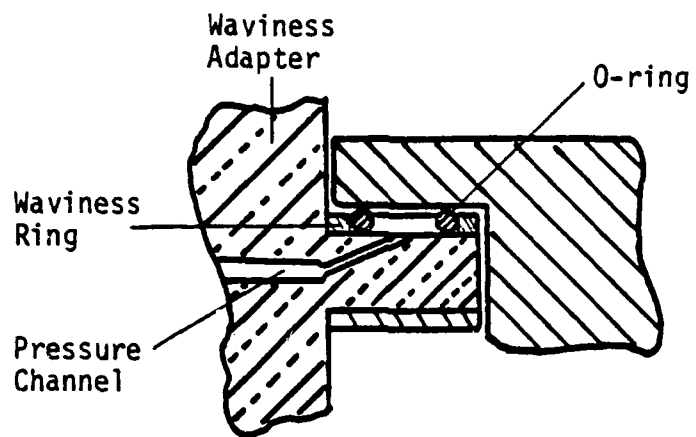


Figure 2-4. Actual Working Cross Section.

By lowering the stiffness another problem developed and hence a design change was needed. Figure 2-5 shows the original O-ring configuration. By lowering the stiffness, the "fingers" were subject to greater deflections which resulted in the O-rings extruding out from under the pad sections. Even with recommended clearance, O-ring extrusion was discovered to be a problem simply because the cyclic motion of the clearance proved to aid extrusion by somewhat of a ratchet mechanism. To remedy this problem, the waviness rings were ground down in thickness to incorporate a spacer, and a piston pad with a smaller O-ring cross section were added (Figure 2-6). This now allowed the "fingers" to flex and the O-rings to be contained without leakage and without extruding.

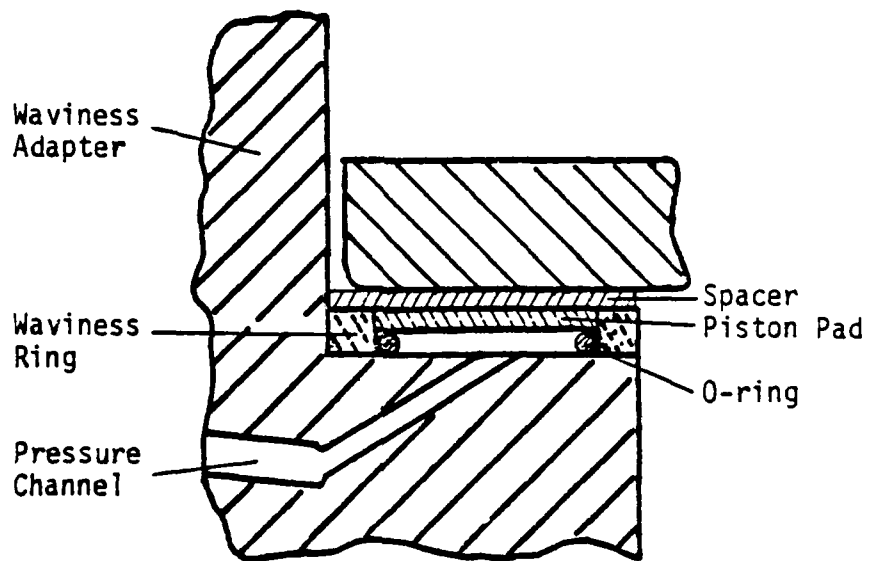
Waviness Measurements--As an aid to understanding the problem with creating sufficient waviness, a means of measuring waviness outside the seal environment was devised. A fixture was made which can pressurize one set of 18 pockets at one time using gas pressure. With one set of pockets under pressure the seal ring was placed on a precision rotary table. The table rotates and the stylus of the surface analyzer was placed so as to read axial displacement near the O-D of the carbon. The signal was digitized and a Fourier analysis of waviness components was made. Using this technique, the nine-wave component was easily picked out. Table 2-1 shows the results of the many measurements taken to analyze the waviness problem.

Test Results--A series of six seal tests were run with the new nine-wave seal. The tests occurred at different points in the modification process mentioned before. As a result, the tests must be carefully compared to one another. Only the performance of Test No. 117 can be evaluated for comparison to theory as a final design seal.



Scale 2:1

Figure 2-5. Original O-Ring Configuration.



Scale 4:1

Figure 2-6. Final O-Ring Configuration with Piston Pads.

Table 2-1

Nine-Wave Amplitude Study  
Waviness Amplitude Near Seal O.D.

Date	$\frac{h}{9}$	P <sub>test</sub>	$\frac{h^*}{9}$ @1500 psi	Conditions
6/28	33	750	66	Close gap epoxy bond, 54-0.030 pads
6/28	59	1390	64	Close gap epoxy bond, 54-0.030 pads
6/16	80	1150	104	Close gap epoxy bond, 18 O-rings only
6/16	65	1050	93	Close gap epoxy bond, 18 O-rings only
6/4	123	1175	157	Groove bottom, 18 O-rings
6/4	129	1200	161	Groove bottom, 18 O-rings
6/2	29	1500	29	Super Glue - after test run, 90 Durometer
6/2	23	1500	23	Super Glue - after test 70 Durometer O-rings
4/7	72	1500	72	Carbon-thick bond epoxy after final x-section modification
4/6	73	1500	73	Carbon-thick bond epoxy after final x-section modification
3/27	54	1500	54	Carbon-thick bond epoxy before final x-section change
3/15	16	1500	16	After first test - thick epoxy

\*Two times the effective waviness pressure in operation.

Table 2-2 shows the results of the tests performed. Test No. 112 (Figure 2-7), first of the series, was run where the carbon insert had a bond line of 0.0254 cm 3M 1836 B/A adhesive. The initial torque levels were quite high, approximately 14 N·m. There was one shutdown at 16 hours into the test due to excessive torque levels. The performance of this test was not as expected.

It was at this point that the stiffness of the seal ring was re-analyzed and as a result, a warping factor was incorporated into the design computer program. Computer results showed that to obtain the desired waviness, the stiffness would have to be reduced by at least 50 percent. The seal was then modified and Test No. 113 (Figure 2-8) was run. This test had the identical start-up behavior of Test No. 112. The test was stopped after only 4.7 hours.

The seal was removed from the test apparatus, traces made of the face, and then a 300  $\mu\text{m}/\text{m}$  convergent taper was lapped into the face. The seal was then replaced in the test apparatus and pressurized to 100 percent pressure for a static test. The seal was left overnight under these conditions. After disassembly, traces showed no convergent taper at all, but rather a flat face condition. This result showed that the epoxy bond creeps and allows the seal face to rotate under pressure. The seal was then re-installed with this flat face condition and run without waviness pressure. The results, Test No. 114 (Figure 2-9), show that the performance was quite similar to the two previous tests. Clearly, any waviness imparted to the seal was not being transmitted to the carbon insert. Other adhesives were investigated and tested to try and remedy this problem.

The carbon insert was machined out and a new one installed using Loctite Speedbonder No. 319 as the adhesive. The bond thickness on the back of the carbon was reduced to zero to stiffen the connection. Test No. 115 (Figure 2-10) was then run. It showed a torque level which was about half of that of

Table 2-2

Waviness Tests  
 $P_{H_2O}$  = 100% pressure, 1800 rpm

P <sup>*</sup> Wave (MPa)	Test No.	Test Duration (h)	Average Face Temperature (°C)	Average Torque (N·m)	Average Leakage (cm <sup>3</sup> /min)
5.2	112	23.5	39.6	11.0	0.00
5.2	113	4.7	39.8	10.2	0.00
0	114	13.2	41.1	11.7	0.00
5.2	115	67.7	39.2	6.4	0.00
5.2	116	47.8	38.8	4.9	0.00
5.2	117	99.8	38.4	2.6	0.19

\*Equivalent waviness pressure



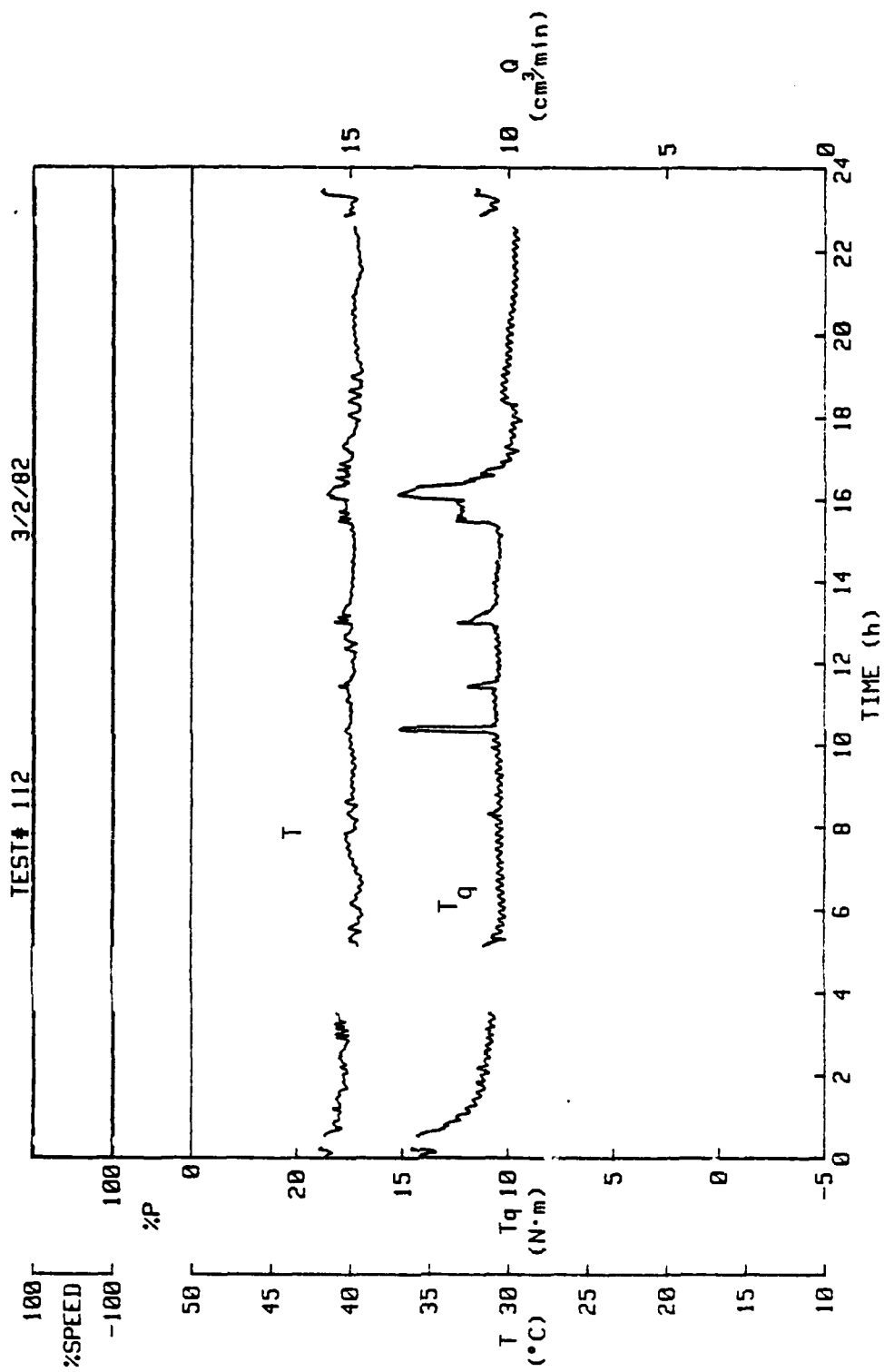


Figure 2-7. Test #112 - Nine-Wave Seal, 100% pressure, 1800 rpm.

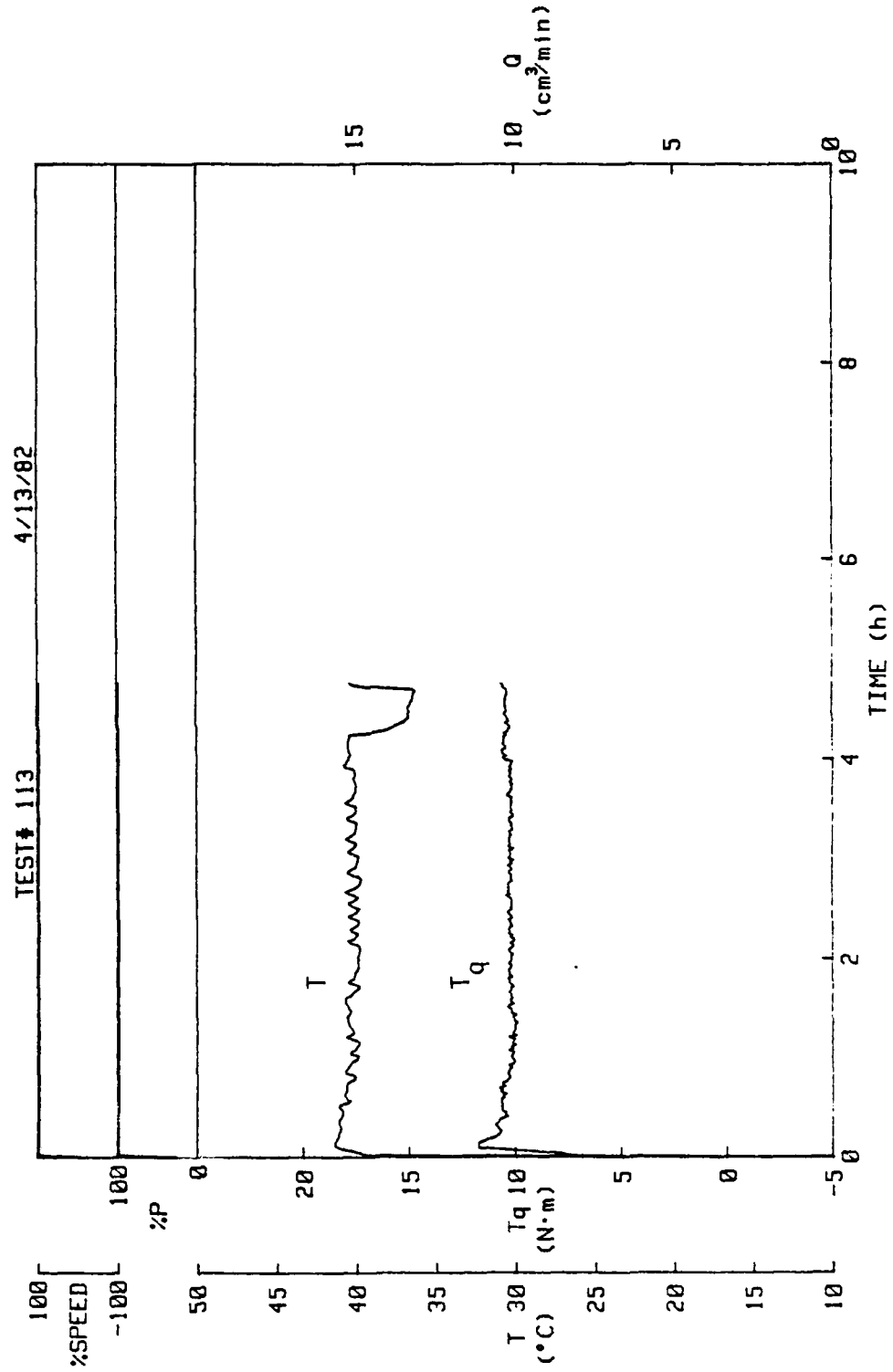


Figure 2-8. Test #113 - Nine-Wave Seal, 100% pressure. 1800 rpm.

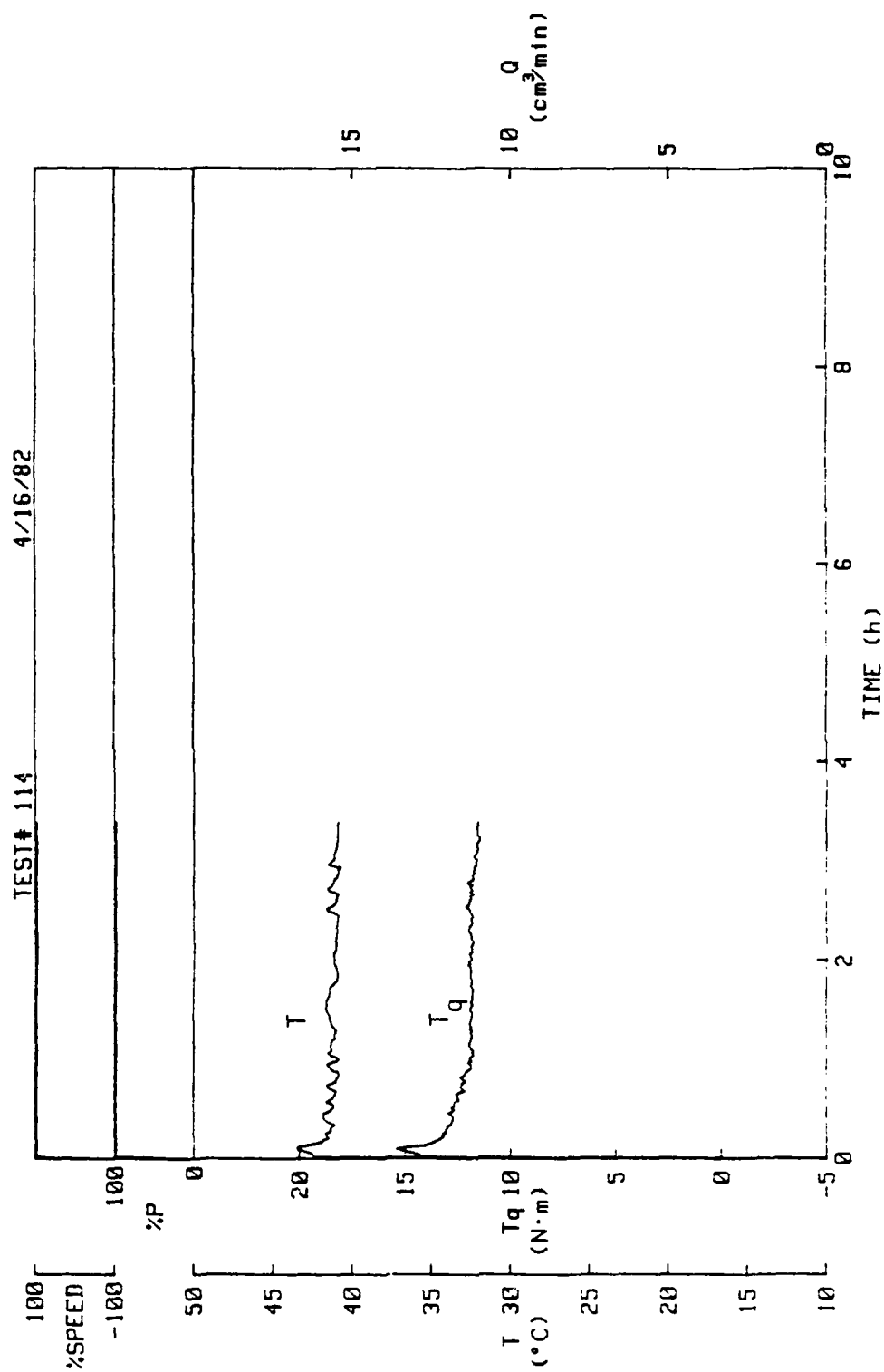


Figure 2-9. Test #114 - Nine-Wave Seal, 100% pressure, 1800 rpm.

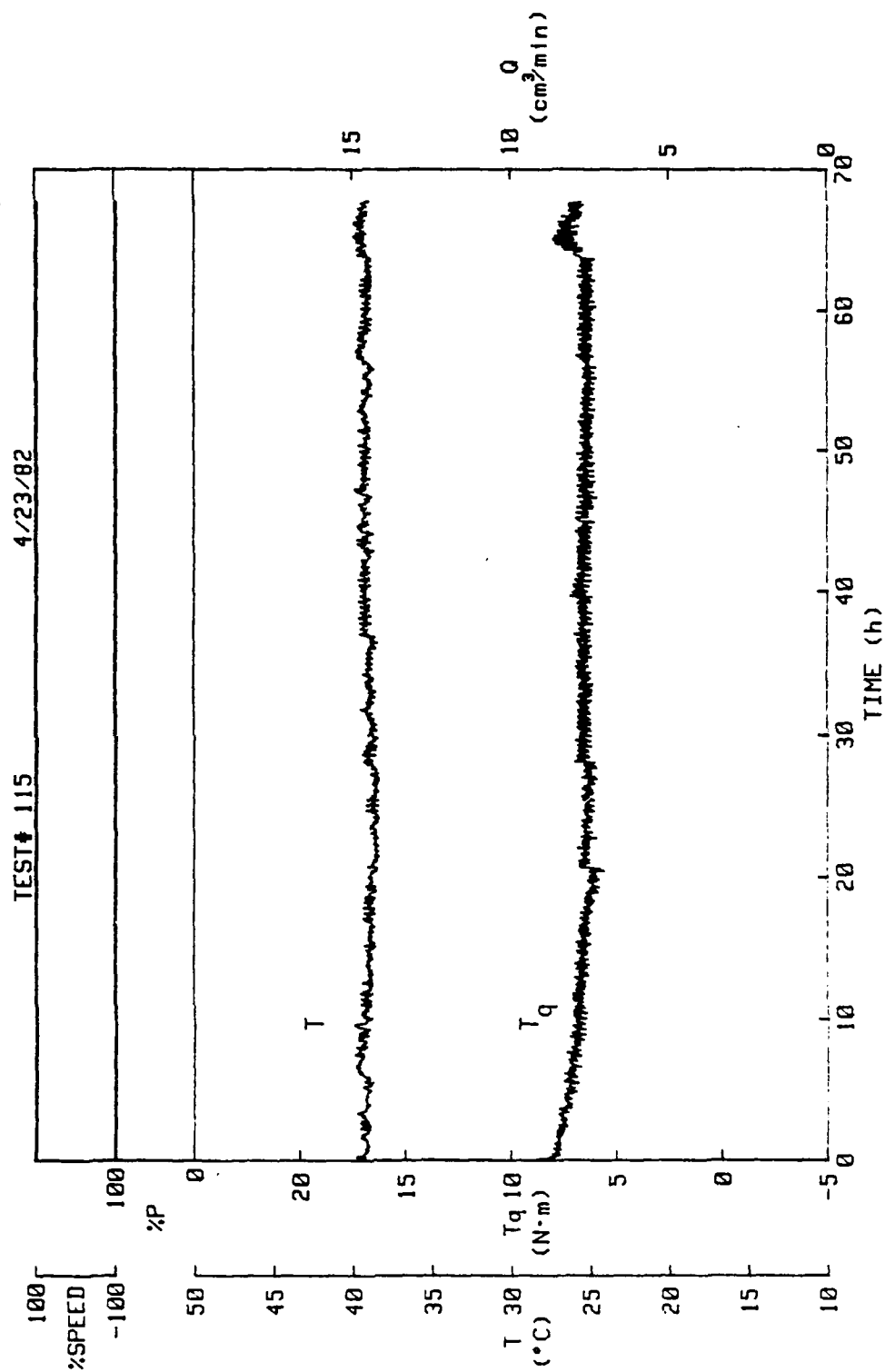


Figure 2-10. Test #115 - Nine-Wave Seal, 100% pressure, 1800 rpm.

previous tests. However, there was no leakage and some was to be expected. The test was ended after 47 hours and the carbon insert was again machined out. There were indications that incomplete bonding had occurred between the seal and insert at certain locations, mainly on the outside diameter.

The next step was to reduce the inner and outer diametrical clearances around the carbon insert. This was accomplished by making two stainless steel spacer rings for the inner and outer diameters. The rings were 0.0254 cm thick and as a result made the carbon insert a slight press fit into the seal. Because of the press fit another type of adhesive was needed. Loctite Superbonder No. 420 was used because of its low viscosity properties and post assembly application ability. The seal was bench tested for waviness and then re-installed in the test apparatus. Test No. 116 (Figure 2-11) was run for more than 47 hours and showed a performance similar to Test No. 115. Some leakage had occurred between 4-1/2 hours and 14-1/2 hours into the test but at no other time.

The test was stopped after the torque readings became erratic, varying as much as 5 N·m. The carbon ring was machined out and again showed indications of incomplete bonding, this possibly being the result of water weakening the adhesive strength. It was decided to test the strengths of the various adhesives used by independent experiment. Table 2-3 shows the results. The results show clearly that the Loctite Superbonder No. 420 is weakened by water contamination and that 3M 1838 B/A was the strongest even after soaking in water.

Based on the previous findings a new carbon insert was cemented into the seal along with the stainless steel spacer rings using the 3M 1838 B/A adhesive. Because of the high viscosity of the epoxy, installation required careful application of the epoxy and pressing in the carbon. The O-rings in the pressure adapter were also modified. Because of the reduced

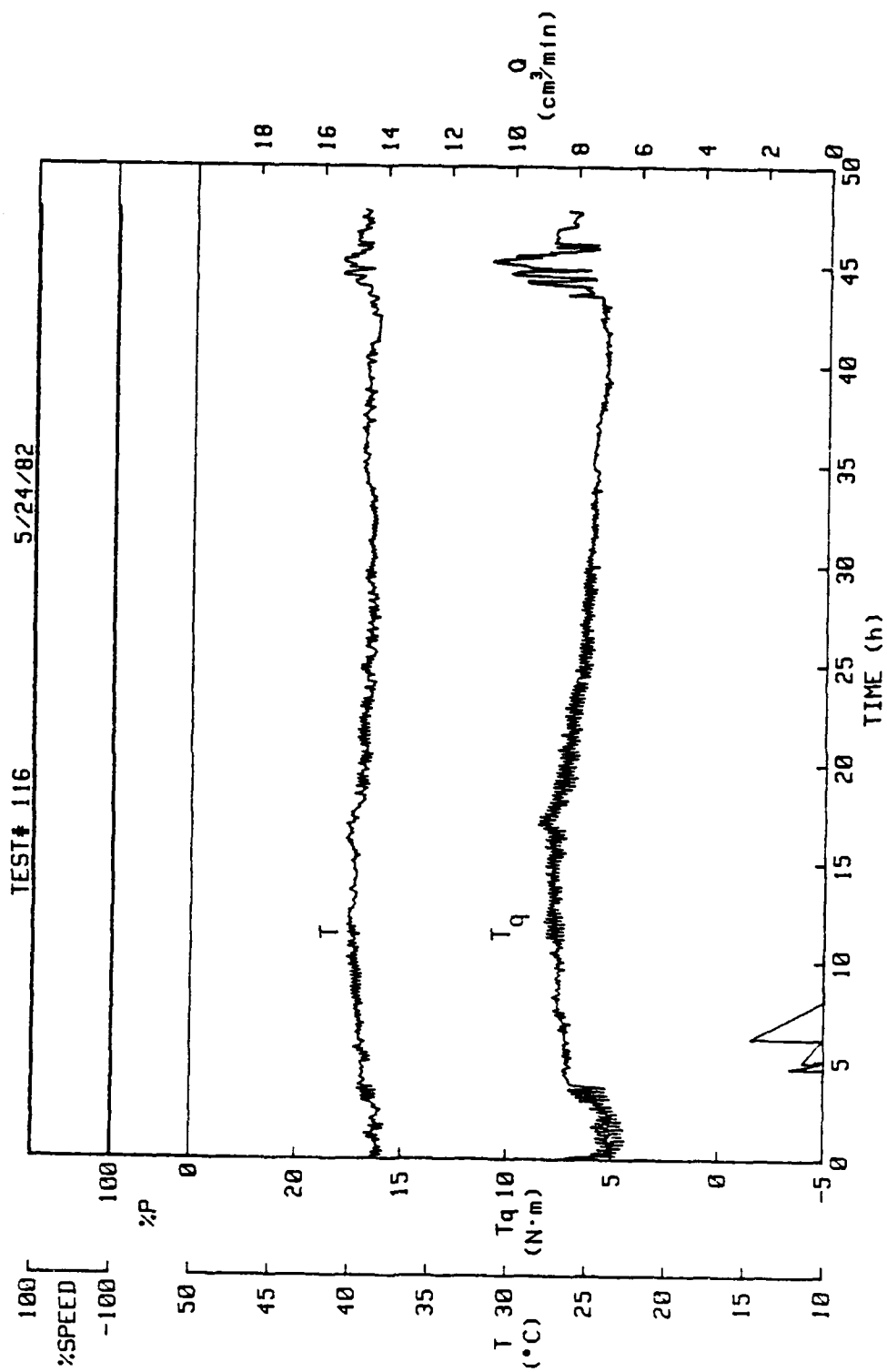
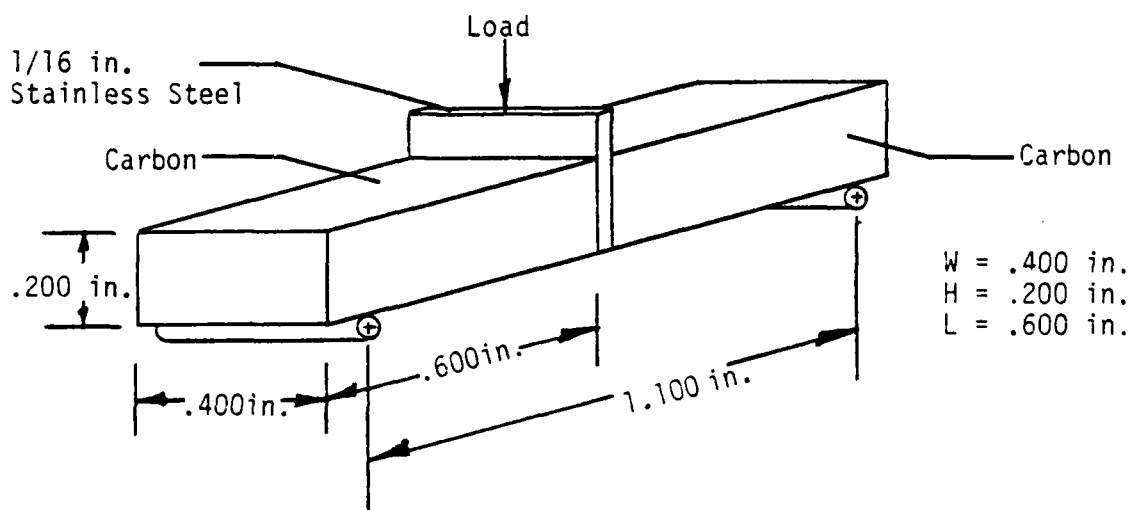


Figure 2-11. Test #116 - Nine-Wave Seal, 100% pressure, 1800 rpm.

Table 2-3

## Carbon to Stainless Steel Bond Strength Tests

Test Specimen	Adhesive	Load at Failure (lb)	Comments
1	Loctite 420 (Super bonder)	3.45	Under static line water pressure for 2 days--clean break at glue line (one side)
2	Loctite 420 (Super bonder)	6.85	Under water 0 psi for 2 days--clean break at glue line (one side)
3	Loctite 420 (Super bonder)	10.25	Moisture free--clean break at glue line (both sides)
4	Loctite 420	--	Not tested
5	Loctite 319 (Speed bonder)	--	Carbon too porous for adhesion--no bond
6	Loctite 319 (Speed bonder)	--	Same as above
7	3M 1838 B/A Epoxy	29+	No break
8	3M 1838 B/A Epoxy	29.05	Carbon broke (not at glue line)



stiffness, O-ring extrusion was becoming a problem, so modifications were made to use smaller O-rings along with piston pads and Delrin retaining rings. The seal assembly was bench tested for waviness and showed good results.

The seal was re-installed and Test No. 117 (Figure 2-12) started. The test showed a start-up performance similar to Test Nos. 116 and 115. The torque was falling to around 5 N·m when water contamination from the building's air compressor clogged the hydraulic pump to the waviness generator. The test had to be stopped after four hours of operation and the pump cleaned. A temporary separate air compressor to operate all air-assisted devices on the test rig was added. The test was re-started and ran for almost 100 hours. Leakage began about eight hours after start and continued throughout the test at about 0.2 cm<sup>3</sup>/min.

Test 117 shows torque and leakage values approaching those expected for the nine-wave seal design and would seem to indicate that the design is successful on a preliminary basis. However, radial taper measurements after the test show that a much greater than expected radial taper is present in the seal. This question is being investigated at this time.

#### Test Machine Modification

Automatic operation of the test apparatus under variable operating conditions required certain modifications of the test apparatus and control hardware and software. These changes are described here.

Speed Controller--The test apparatus is equipped with a 3700 W (5 Hp) belt type variable speed drive ac motor. Speed change, as equipped from the factory, was accomplished by means of a hand crank which varied belt pulley ratios. By this method, speeds from 380 to 4000 rpm were obtained. However, for the simulated submarine operation test program, it was desired that



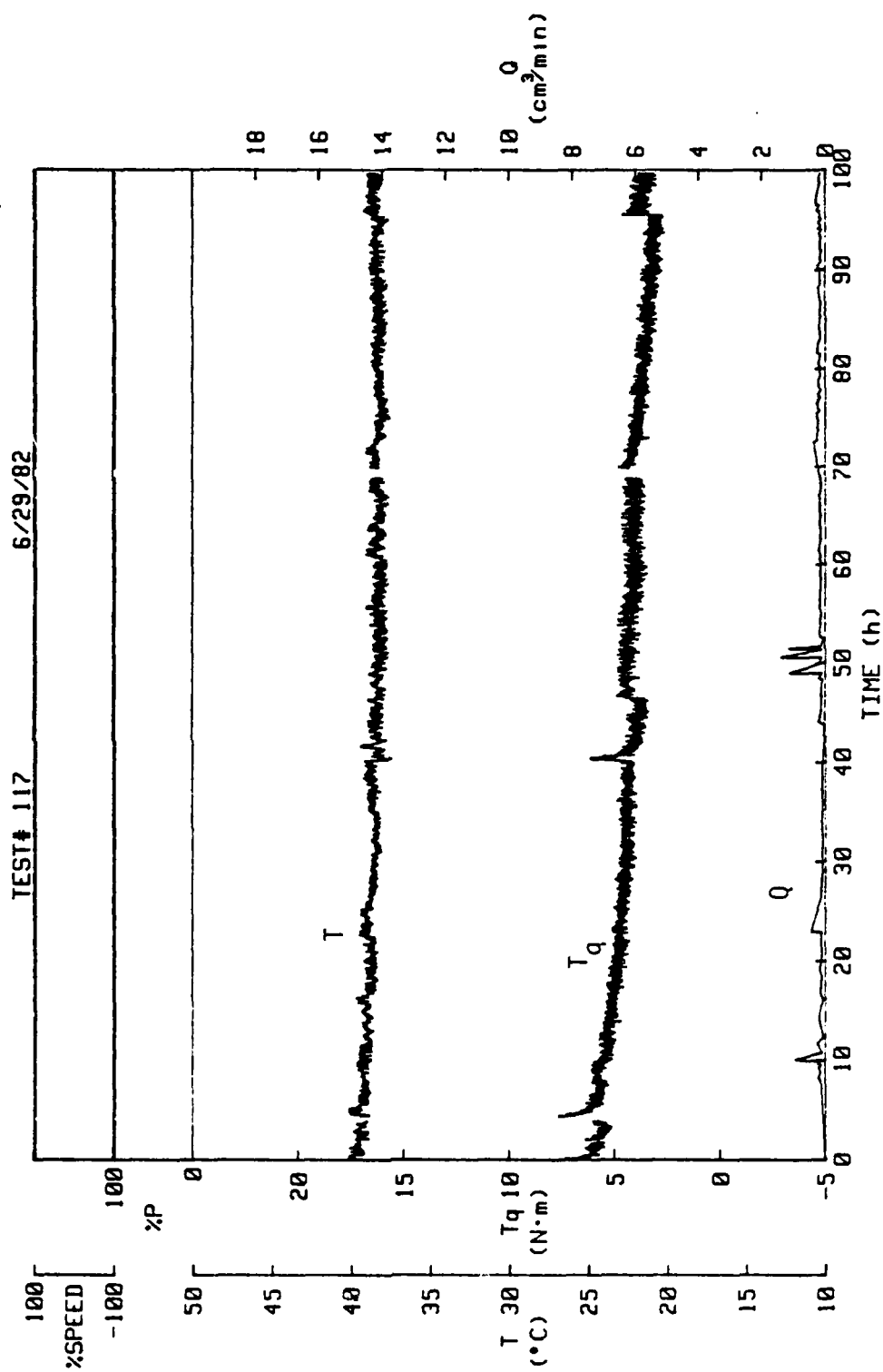


Figure 2-12. Test #117 - Nine-Wave Seal, 100% pressure, 1800 rpm.

the computer system would control the speed, thus providing an automatic operation so the operator could save time and do other things while running a test.

Modification began with the addition of a Superior Electric Co. Slo-Syn stepping motor, Model No. SS 250-1027, and a 2:1 step-down gear ratio in place of the hand crank mechanism. This, coupled with a Slo-Syn ST 1800B stepping motor translator and a H.P. 69335A stepping motor control card for use in the H.P. 6940B multiprogrammer unit, was the basic hardware of the speed control apparatus. Since the stepping motor operates by the input of a pre-programmed pulse train, each pulse of which will rotate the stepping motor shaft by  $1.8^\circ$ , a relationship between stepping motor shaft rotation and the main drive motor rpm was found. Motor speed control was achieved by inputting the desired rpm into the computer which then converts this to the appropriate number of pulses. A data word is sent to the stepping motor control card in the multiprogrammer unit which contains the desired number of pulses. The stepping motor control card in turn outputs the pulse train to the translator which then sends the stepping motor the required number of pulses which in turn changes the ratios of the variable speed drive.

Two other factors had to be considered for complete motor control. The first is the capability of forward and reverse control and the second is minimum speed. Forward and reverse control was accomplished by use of two reed relays on the relay card in the multiprogrammer unit, two DPDT relays and two motor starter control units. Figure 2-13 shows the schematic diagram of the directional control circuit. Directional control is achieved by sending a control word from the computer to the relay card to close either Relay Nos. 9 or 10 depending on the desired motor direction. A desirable feature of this circuit is that it is impossible to energize both forward and reverse motor starter units simultaneously by closing both Relay Nos. 9 and 10 on the

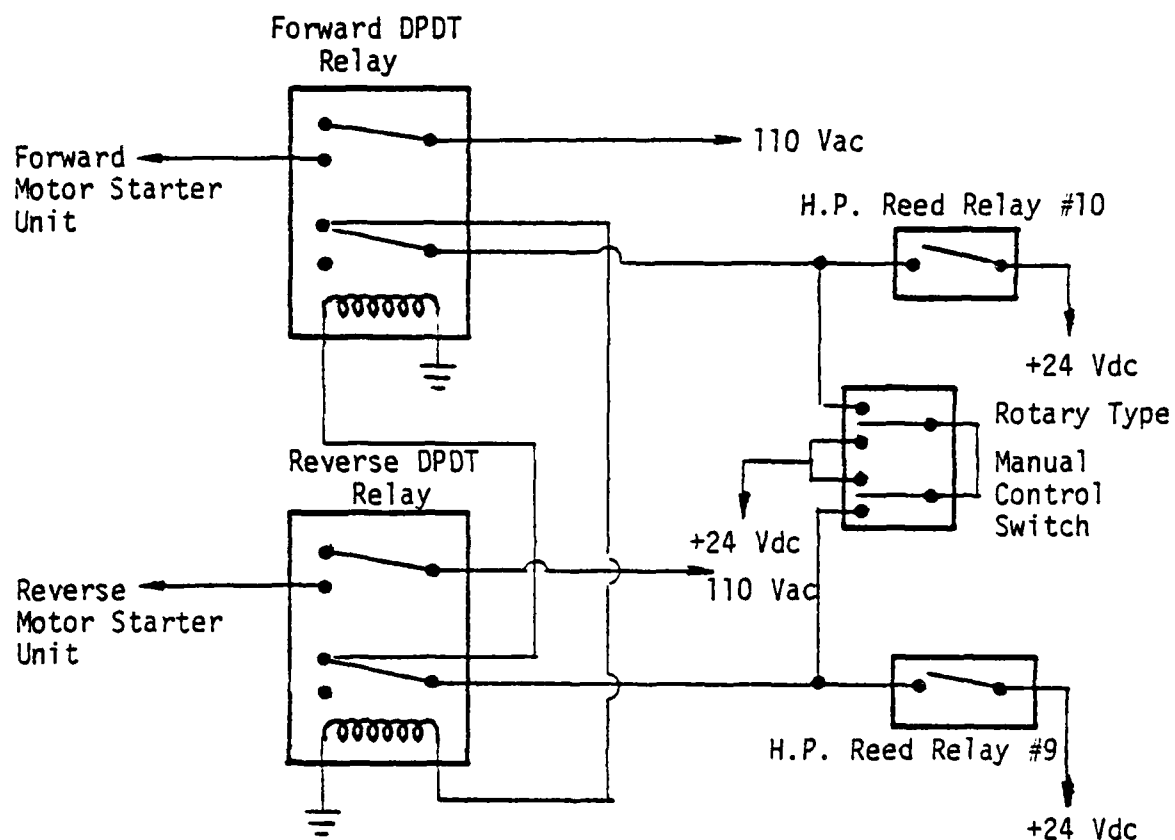


Figure 2-13. Directional Control Circuit.

relay card. A rotary type switch was added so as to obtain manual directional control external to the computer.

A lower minimum speed was obtained by the use of a Reliance Electric variable frequency ac motor speed controller. This unit has the capability of changing the input frequency to the ac motor from 6 to 60 Hz, thereby changing the speed of the motor also. This device used in conjunction with the stepping motor speed controller now gives a range of speed control from approximately 40 to 4000 rpm.

Pressure Controller--Another area of computer control needed for the test program is that of sealed water pressure regulation. The test apparatus is equipped with an accumulator to dampen out pressure surges. The accumulator is shown in Figure 2-14. When the accumulator is charged with nitrogen gas to 75 percent of the water pressure, then proper surge control is obtained in that 75 percent of the original accumulator volume remains as gas when the liquid and gas pressures become equal. But once charged, if the water pressure is changed significantly, up or down, then the diaphragm will also change location and as a result poor performance of the accumulator can be expected. To operate at variable pressure conditions, it therefore becomes necessary to be able to detect the location of the diaphragm and correct its position when needed for optimum performance.

Figure 2-14 shows how this positioning capability is obtained. When the diaphragm is displaced either up or down due to changes in water pressure, the guide rod will move the permanent magnet also. When the permanent magnet is moved so that it is adjacent to one of the magnetic reed switches, the magnetic field induces a contact closure in that reed switch. By correctly positioning the reed switches, an upper and lower bound for diaphragm location can be set along with a sufficient dead zone of operation and an optimum diaphragm position.

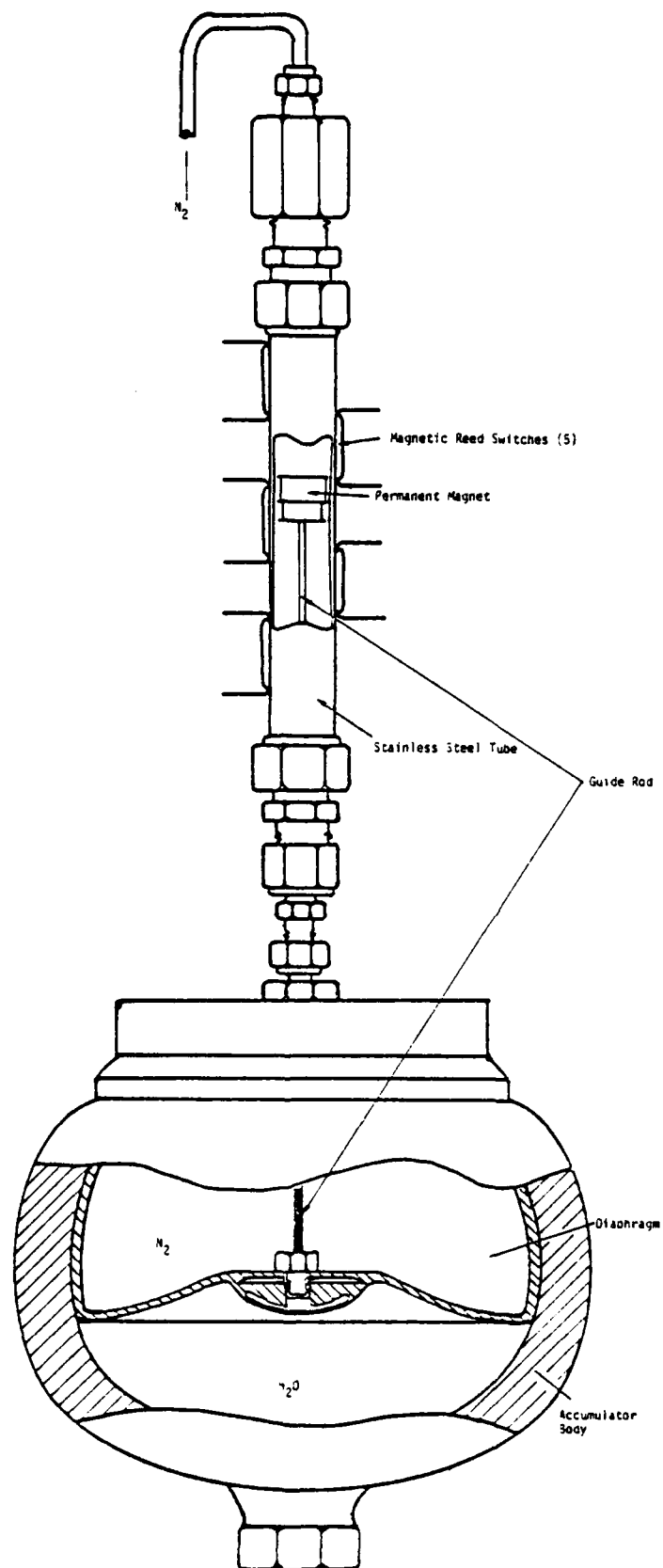


Figure 2-14. Diaphragm Positioning Device.

The accumulator diaphragm positioning circuit is shown in Figure 2-15. When the permanent magnet is adjacent to the No. 3 reed switch then the engaging coil of both Nos. 6 and 7 dual coil relays will be energized, thereby setting the switch in both relays to the engaged position. If the diaphragm rises so that it is next to either the No. 2 or 1 reed switch then the disengaging coil of No. 6 dual coil relay is energized which then registers an interrupt on Channel No. 6 of the interrupt card in the multiprogrammer unit. The computer recognizes that this interrupt indicates a high diaphragm and in turn operates two solenoid valves to let water out of the system and add  $N_2$  to the top side of the accumulator which will result in lowering the diaphragm. Once the disengaging coil is energized, it will remain so sending an interrupt to the multiprogrammer unit. This will continue until the diaphragm moves down so that the magnet again closes the No. 3 reed switch and as a result energizes the engaging coil and discontinues the interrupts. Likewise, a low diaphragm will register an interrupt on Channel No. 5 of the interrupt card. This prompts the computer to cause the pump to pump more water into the system and let some  $N_2$  out by means of another solenoid valve. This set-up therefore allows for pre-programmed pressure changes during testing and ensures that the optimum diaphragm positioning is obtained at any pressure.

Misalignment Control--Another feature required for the simulated submarine operation test program is a variable tilt and offset of the seal. This was accomplished by the set-up shown in Figure 2-16. A Slo-Syn SS 250-1027 stepping motor and gear reduction arrangement is connected to the shaft of the worm which drives the waviness cylinder. This waviness cylinder has a tilt and offset machined into it which begins just right of the end plate. To the left of this point the waviness cylinder is concentric with the shaft. Using the computer to pulse the stepping motor, three different tilts with corresponding offsets, as

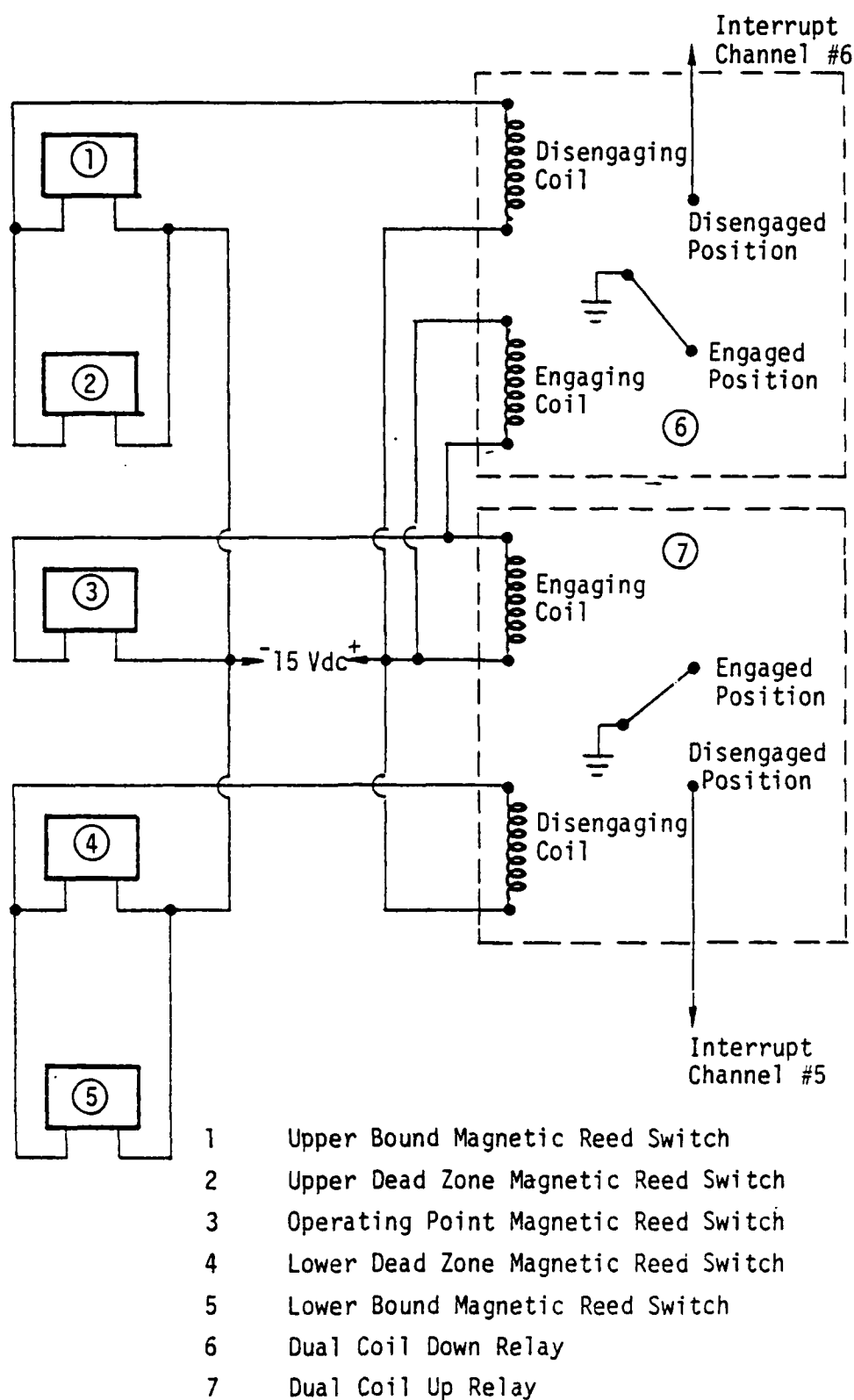


Figure 2-15. Accumulator Diaphragm Positioning Circuit.

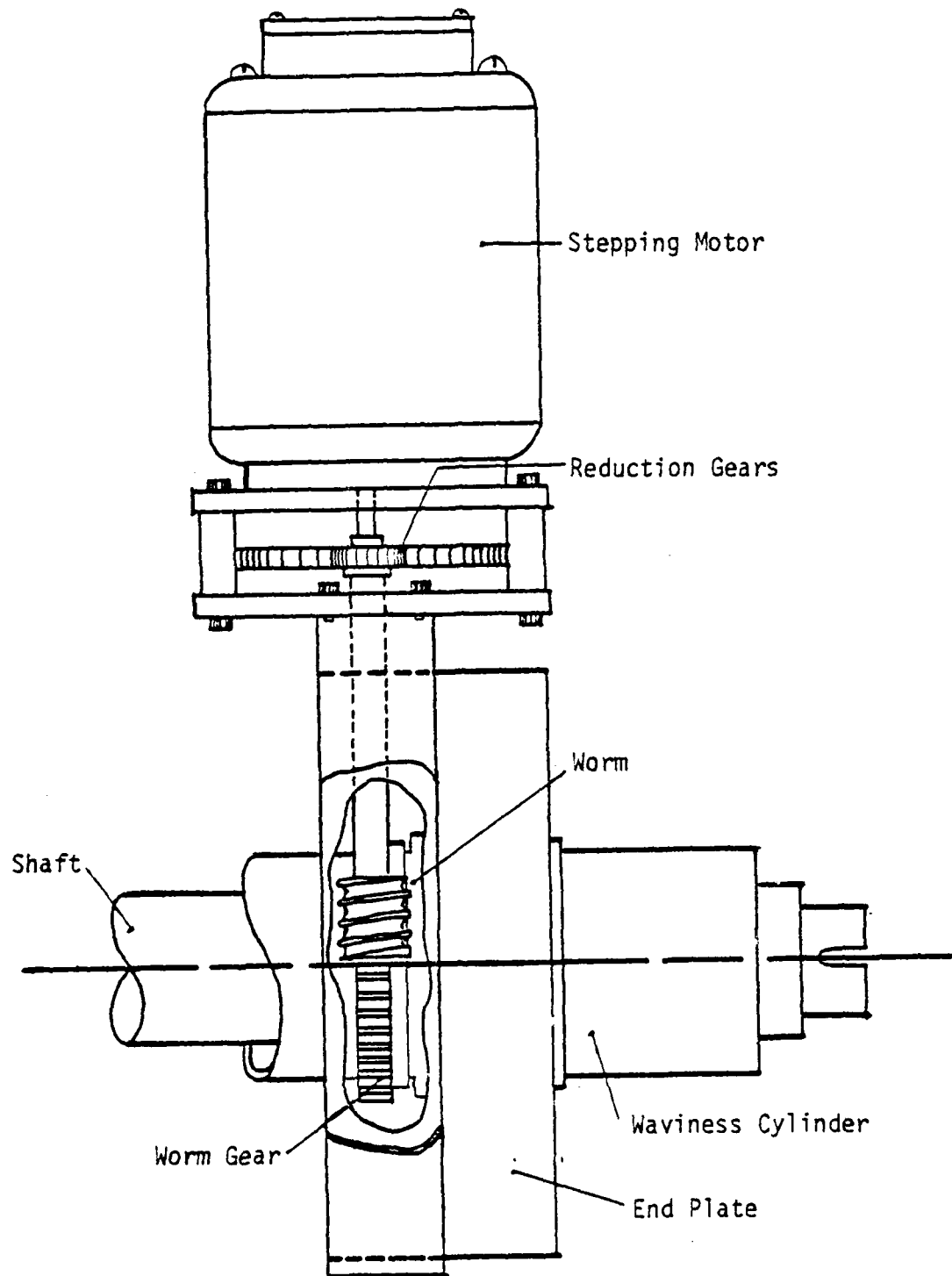


Figure 2-16. Tilt and Offset Drive.



specified by the test program, can be obtained. Details of the geometry related to this procedure are described in Chapter 3.

#### Seventy-Five Percent Balance Ratio Test

Test No. 111, Appendix A, was run at a balance ratio of 0.75 and zero initial taper so as to obtain additional wear rate data. The operation of this test was somewhat similar to previous tests run under the same conditions. One difference was that leakage did not occur until some 390 hours into the test. Similar tests showed leakage initially. Also, whereas the average torque level was comparable to other tests, there were large fluctuations associated with it in the initial start-up. These became lower as the test proceeded. The test ran much more smoothly after 48 hours of operation and up to approximately 412 hours where the torque levels again began to fluctuate. The test was eventually shut down due to high torque at 494 hours.

This type of operation at the end of the test is comparable with Test Nos. 75 and 76 [5], and final radial traces of the carbon face also showed a taper of about  $-1000 \mu\text{m/m}$ . Test No. 75 had a final taper of  $-710 \mu\text{m/m}$  and Test No. 76  $-982 \mu\text{m/m}$ . This behavior is to be compared to that for Test 66 which was also for a zero initial taper. Test 66 started up with low torque and some leakage and continued about the same for over 100 hours. Clearly, Test 111, which was outwardly identical, started out in a different mode. Test 66 started out taking advantage of a thermal taper which never wore off during the test. Test 111 never got into this mode. It somehow got started in a mode where it wore rapidly into a nonleaking parallel face condition with a high but worn away thermal taper. The last hours of operation of Test 111 show that this mode is not really stable but can lead to high friction, high wear operation. The intriguing question which requires still more testing to answer is why this test started in the high friction mode and Test 66 started in a low friction mode.

## CHAPTER 3

### NINE-WAVE SEAL DESIGN

#### Criteria

In this chapter the design procedure for the nine-wave seal is presented in detail. The criteria set forth for the design are as follows.

- 1) Very low wear--10 year life,
- 2) Moderate to low leakage--leakage to be consistent with time,
- 3) Low friction (to ensure low thermal distortions)
- 4) Operation in the 500 psi, 1800 rpm at 4 inch mean diameter range,
- 5) Operation in seawater,
- 6) Variable speed, pressure, and distortion as functions of time to conform to simulated submarine operation,
- 7) Seal components themselves must be reliable to be compatible with 1) above,
- 8) Use of carbon insert in a metal ring to be compatible with large scale design.

To satisfy the above conditions using a wavy seal, certain additional criteria become apparent. Some of these are based on conclusions from the previous report [5] and a paper on this subject [14].

1) The seal rings must be designed so that the pressure caused rotation is zero. This is essential in a seal where operating pressure changes frequently. If pressure caused rotation occurs, then the seal face must wear to a new profile at each operating pressure.

2) Nine waves must be imposed on the seal. References [5] and [14] show that the natural pressure variation on the seal

face produces a tilt angle which is dependent on operating conditions. Such changes will lead to accelerated wear and must be minimized. By increasing the number of waves the relative tilt stiffness of the seal increases such that operating conditions no longer produce a change in tilt which is significant relative to the applied tilt. At nine waves the tilt changes only a few percent with variable operating conditions.

3) The centroid of the cross section must be located so that the sealing radius is near the inside radius  $r_i$ . This ensures that both minimum leakage and maximum load support will occur. This imposes a severe geometrical constraint on the cross section.

4) So that the seal may easily comply with the mating ring at the lower harmonics and so that the needed waviness can be reasonably imposed, the cross sectional properties (moments of inertia) must be maintained at low values (no stiff rings). This requirement restricts the size of the cross section.

#### Assumed Configuration

Figure 3-1 shows the assumed basic configuration for the design. Several other configurations were evaluated, but the one shown appeared to be the best. There are 54 moment arms attached to the seal cross section, 27 on the inside and 27 on the outside. This number represents 3 sets of 18 arms (see details in previous chapter). Eighteen arms (nine inside and nine outside) are required to produce one set of nine waves. The force is applied hydraulically as shown. O-rings create a pressurized pocket, and this pressure acts on the pad. The carbon is bonded into a metal ring. The centroid is shown to be located near the inside radius of the ring. Since the forces on the pads alternate in direction, there is no net radial force due to hydraulic pressure acting on the ring. The seal support must drop down after the secondary O-ring so that some significant amount of

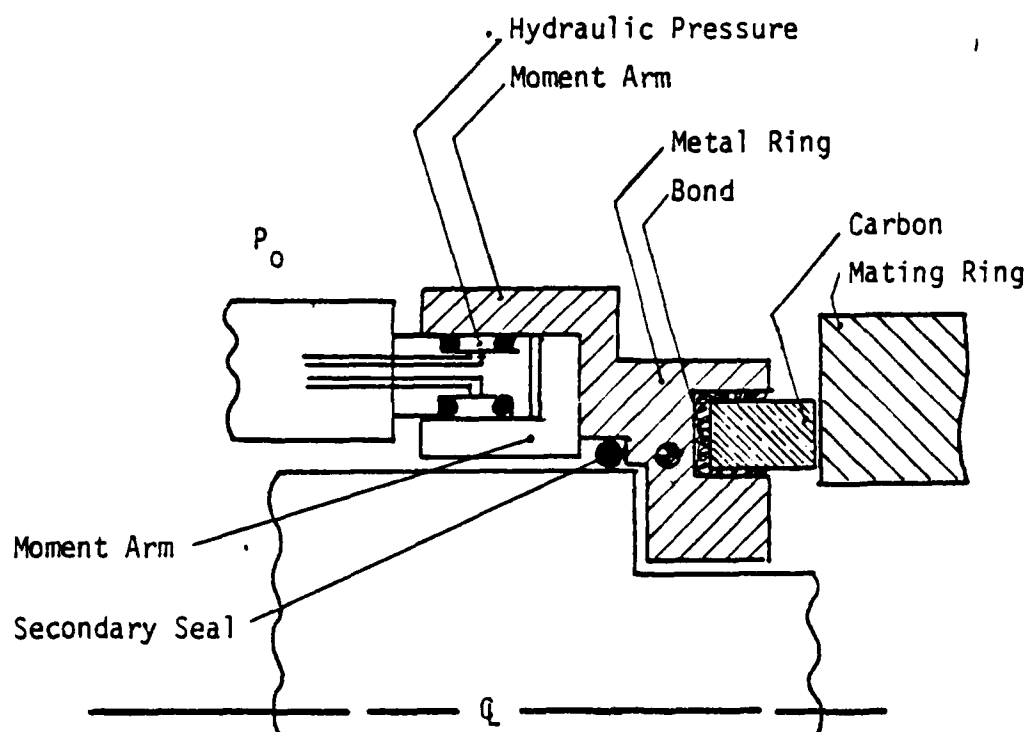


Figure 3-1. Assumed Nine-Wave Seal Configuration.

material can be attached to the cross section below the balance radius in order to lower the centroid.

The diameter of the existing test vessel contains the outside diameter of the seal assembly, and this restricts the size of the moment arms. The fact that the hydraulic pressure is applied on the pressurized side of the seal creates a problem in that the hydraulic pressure must always be above the sealed pressure. If it is not, the O-rings can collapse inward as shown. Even if this problem is overcome by inserts inside the O-rings, it can be shown that a force discontinuity will occur as pressure drops below the sealed pressure. According to theory previously developed [5], the average hydraulic pressure (which is influenced by the sealed pressure) does not affect the amplitude of the wave, so having the hydraulic pads in the sealed pressure environment causes no theoretical problem.

It was immediately recognized that because of all of the O-ring seals operating at high pressure, no significant corrosion of the surfaces could be tolerated. It was learned that Inconel 625 was one of the few alloys that would not pit in seawater, so it was chosen for the metal parts of the seal assembly. Inconel 625 is known to be difficult to machine compared to stainless steel alloys and monel, but it appeared to be the only acceptable alloy from the corrosion standpoint.

#### Seal Ring Design Solution

Figures 3-2 and 3-3 show the details of the seal ring itself. Many of the dimensions shown had to be selected based on the criteria discussed. The details of this procedure will now be described.

- 1) Fix certain parameters (see Figures 3-2 and 3-3)

$K = 1.005$  (explained later)

$R_i = 1.9$  in. (consistent with test machine)

$R_b = 1.9$  in. (balance ratio = 1.0)

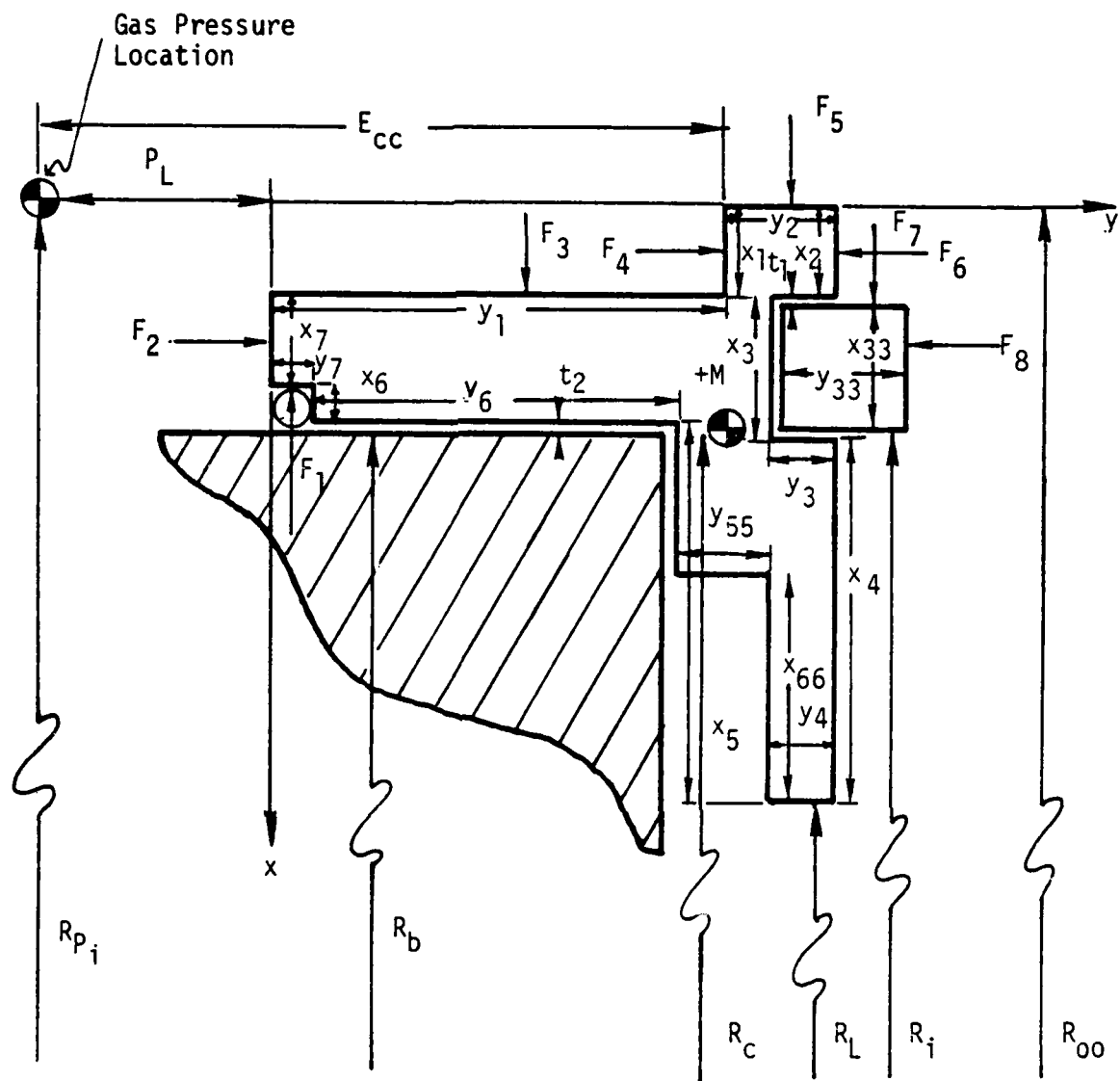


Figure 3-2. Assumed Seal Ring and Moment Arm Geometry.

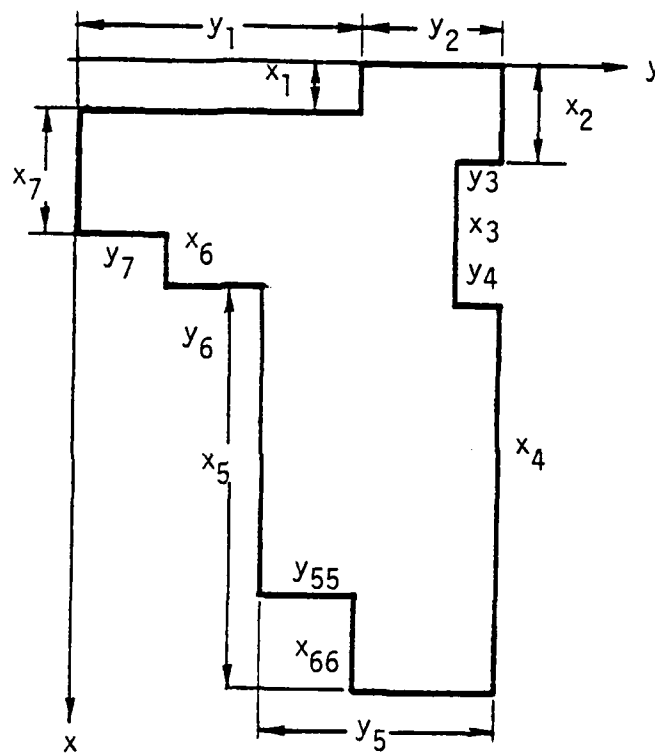


Figure 3-3. Assumed Cross Sectional Geometry.

$x_3 = 0.1875$  in. (face width is equal to existing test seal)

$y_3 = 0.125$  in. (nose height)

$t_1 = 0.010$  in. (bond thickness)

$t_2 = 0.003$  in. (clearance)

$x_6 = 0.051$  in. (required for proper squeeze) (3-1)

$y_7 = 0.080$  in.

$p_0 = 500$  psi (seal working pressure for design purposes)

$n = 9$  (number of waves)

$p_g = 1000$  psi (equivalent gas pressure for calculation. Using a moving wave produced by three sinusoidal with time waves requires that each of the waves have a pressure amplitude of 667 psi)

$E = 31 \times 10^6$  psi (Inconel 625)

$\mu = 0.3$

$$G = \frac{E}{2(1 + \mu)}$$

2) Choose the remaining set of dimensions. These will be varied to obtain the desired design. They are:

$x_1, x_2, x_6, y_1, y_2, y_3, y_5, y_6$  - defined as shown in Figure 3-2

$d$  - pressure pad O-ring diameter (3-2)

$\phi$  - desired seal from tilt amplitude

3) Assuming a value for  $R_L$ , calculate section properties  $\bar{x}, \bar{y}, r_c$ , and  $I_x$ . These are calculated by breaking the cross section up into rectangular subparts as shown in Figure 3-3 using the parallel axis theorem.

4) Calculate  $J_\theta$  (torsional stiffness). Torsional theory requires that the following partial differential equation be solved for the cross section.



$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -2, \quad (3-3)$$

$$\psi = 0 \text{ on boundary.} \quad (3-4)$$

Once  $\psi$  is found, then

$$J_{\theta} = 2 \iint_A \psi \, dA. \quad (3-5)$$

This problem was solved numerically by dividing up the cross section into a grid as shown in Figure 3-4. Note the grid pattern corresponds to the exact boundaries of the cross section. This causes unequal spacing of the grid points. Equation (3-5) was solved using finite difference formulas which account for unequal spacing. The system of equations was solved quickly by relaxation.

5) Calculate  $A = EJ_x/GJ_{\theta}$ . Using this and the previously derived formulas [5],

$$\phi = \frac{m_{\theta_0} r_c^2}{EJ_x} \frac{1 + An^2}{(n^2 - 1)^2}, \quad (3-6)$$

$$m_{\theta_0} = \frac{p_g d R p_i e}{r_c} \sin\left(\frac{nd}{2 R p_i}\right), \quad (3-7)$$

calculate the required  $m_{\theta_0}$ . Given

$$R p_i = R_L + x_5 + x_6 + x_7/2 \quad (3-8)$$

which places the pressure force near the middle of the back edge of the seal ring, calculate the moment arm  $e$  needed to produce the required tilt.

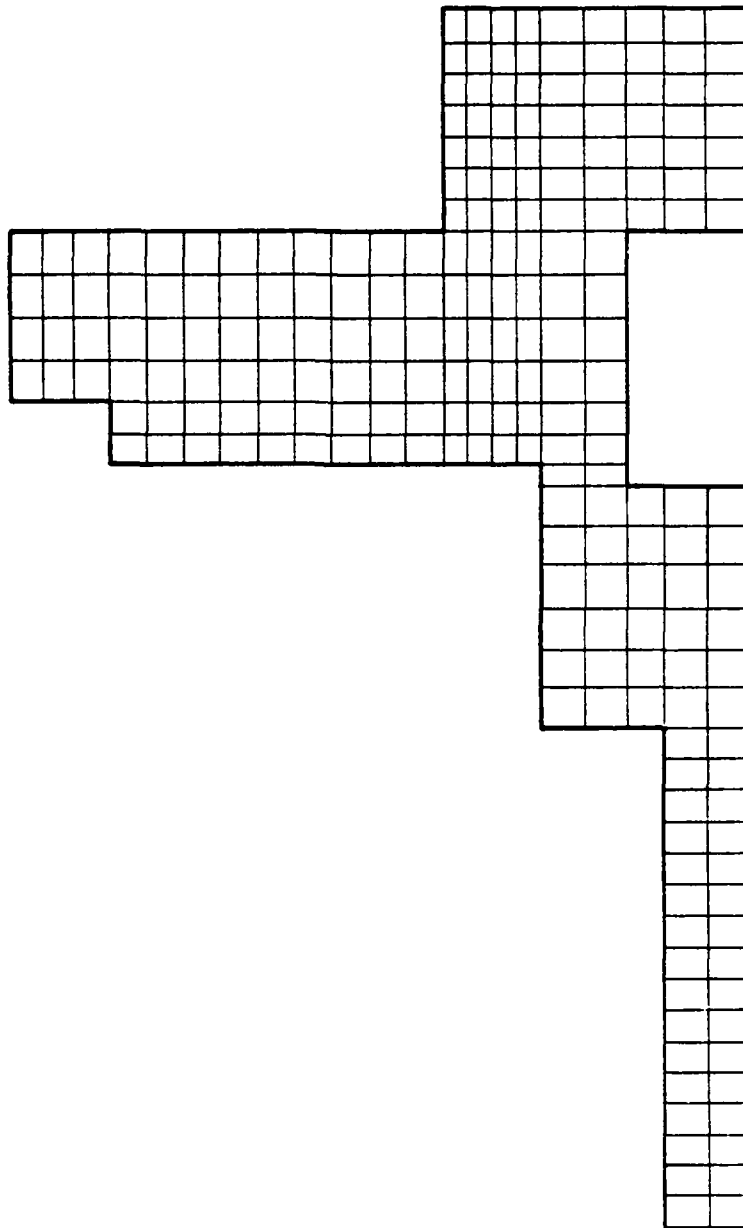


Figure 3-4. Finite Difference Mesh for Calculating Torsional Stiffness.

6) From before [5], calculate

$$v = \frac{m_{\theta_0} r_c^3}{EJ_x} \frac{(1 + A)}{(n^2 - 1)^2} \quad (3-9)$$

and

$$r_{\text{contact actual}} = \frac{v}{\phi} + r_c \quad (3-10)$$

7) To insure proper sealing but also maximizing load support, the contact radius must be just inside the inside radius of the seal or

$$r_{\text{contact desired}} = Kr_i \quad (3-11)$$

K was defined previously as a number greater than one. Now compare:

$$r_{\text{contact actual}} \text{ to } r_{\text{contact desired}} \quad (3-12)$$

If they are significantly different, adjust  $R_L$  using a root finding technique. Go back to Step 3 and repeat all of the calculations until Equation (3-12) is satisfied.

8) Calculate  $I_y$ . Calculate the moments due to pressure as shown in Figure 3-2. Calculate pressure caused rotation

$$\phi_{\text{pressure}} = \frac{m_{\theta} r_c^2}{EJ_x} \quad (3-13)$$

where  $m_{\theta}$  is the moment about the centroid in N·m/m of centroidal circumference.

9) The above program is rerun adjusting the input parameters (3-2) until pressure caused rotation (3-13) is made sufficiently small.

The computer program written to perform the above calculations is included in Appendix B.

### Final Design

After many trials and alterations in configuration, the final design selected is described by the output from the program shown as Table 3-1. Figure 3-5 shows the seal cross section which corresponds to these dimensions. Figures in Chapter 2 show other details of the seal.

### Predicted Performance

Using programs developed previously [5], performance of the new seal design was predicted. Figures 3-6 and 3-7 were obtained using a program which neglects elastic deflection of the seal rings brought about by face pressure itself. As discussed previously at  $n = 9$  this deflection is negligible. Figure 3-6 shows a significant friction torque at the lower speeds with a significant reduction at increased speed. Even at zero speed, predicted torque is about one half of what it would be without any waviness. Some wear is expected at the lower speeds because the fraction of the load supported by fluid pressure drops to 75 percent at 200 rpm. However, because the speed is low where the fluid pressure load support is low, the amount of wear will be minimized. At the higher speeds, the predicted fluid pressure load support becomes 100 percent due to hydrodynamic effects as is suggested by the decreasing torque with increasing speed curve.

Leakage is shown in Figure 3-7. Leakage is predicted to be less than 1.0 cc/min under all test conditions. A large value of  $\phi$  increases leakage considerably but reduces torque very little.

Table 3-1  
Seal Design

$\phi = 320 \times 10^6$	
$d = 0.254 \text{ in.}$	
$x_1 = 0.2100 \text{ in.}$	$y_1 = 0.3800 \text{ in.}$
$x_2 = 0.2100 \text{ in.}$	$y_2 = 0.2200 \text{ in.}$
$x_3 = 0.2075 \text{ in.}$	$y_3 = 0.1000 \text{ in.}$
$x_4 = 0.5010 \text{ in.}$	$y_4 = 0.1000 \text{ in.}$
$x_5 = 0.5140 \text{ in.}$	$y_5 = 0.1700 \text{ in.}$
$x_6 = 0.0510 \text{ in.}$	$y_6 = 0.3500 \text{ in.}$
$x_7 = 0.1435 \text{ in.}$	$y_7 = 0.0800 \text{ in.}$
$R_{pi} = 2.4840 \text{ in.}$	
$R_L = 1.3890 \text{ in.}$	
$\bar{x} = 0.3518 \text{ in.}$	
$\bar{y} = 0.3943 \text{ in.}$	
$I_x = 0.00542 \text{ in.}^4$	
$I_y = 0.00916 \text{ in.}^4$	
$J_\theta = 0.00176 \text{ in.}^4$	
$A = 7.86$	
$v = 8.7 \times 10^{-6} \text{ in.}$	
$e = 0.953 \text{ in.}$	
$r_c = 1.9557 \text{ in.}$	
$\phi_{\text{pressure}} = 6.8 \times 10^{-8} \text{ in./in.}$	

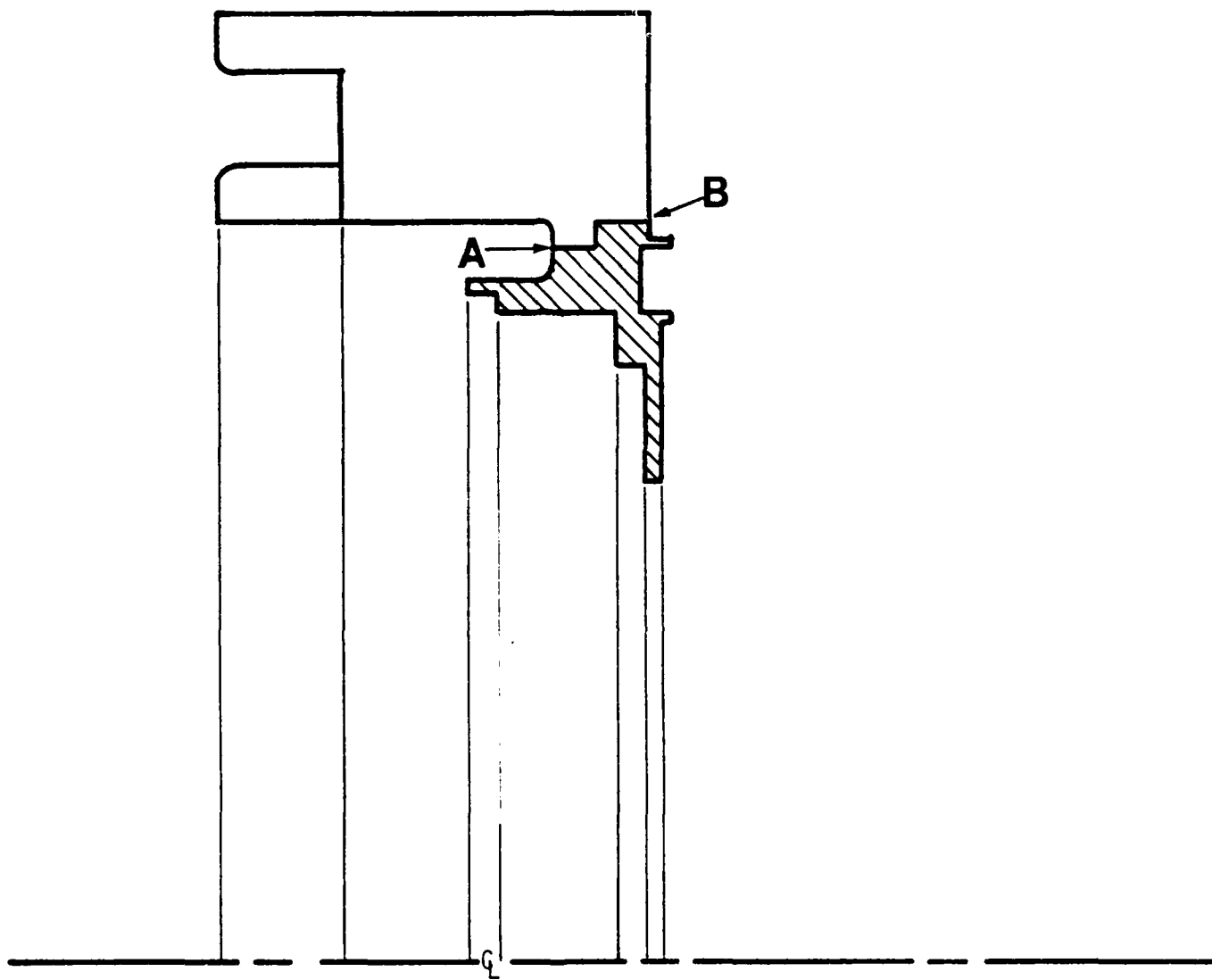


Figure 3-5. Actual Seal Cross Section.

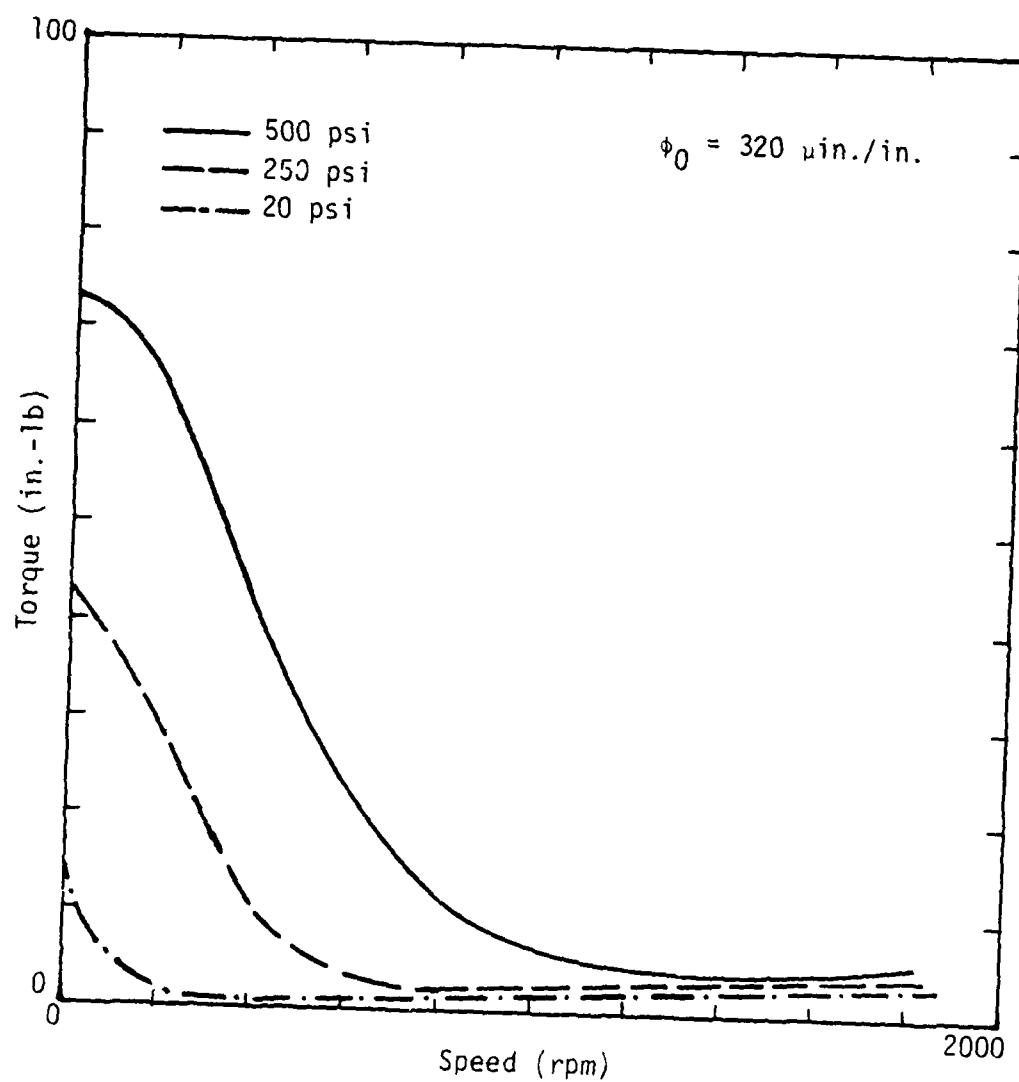


Figure 3-6. Predicted Performance - Torque.

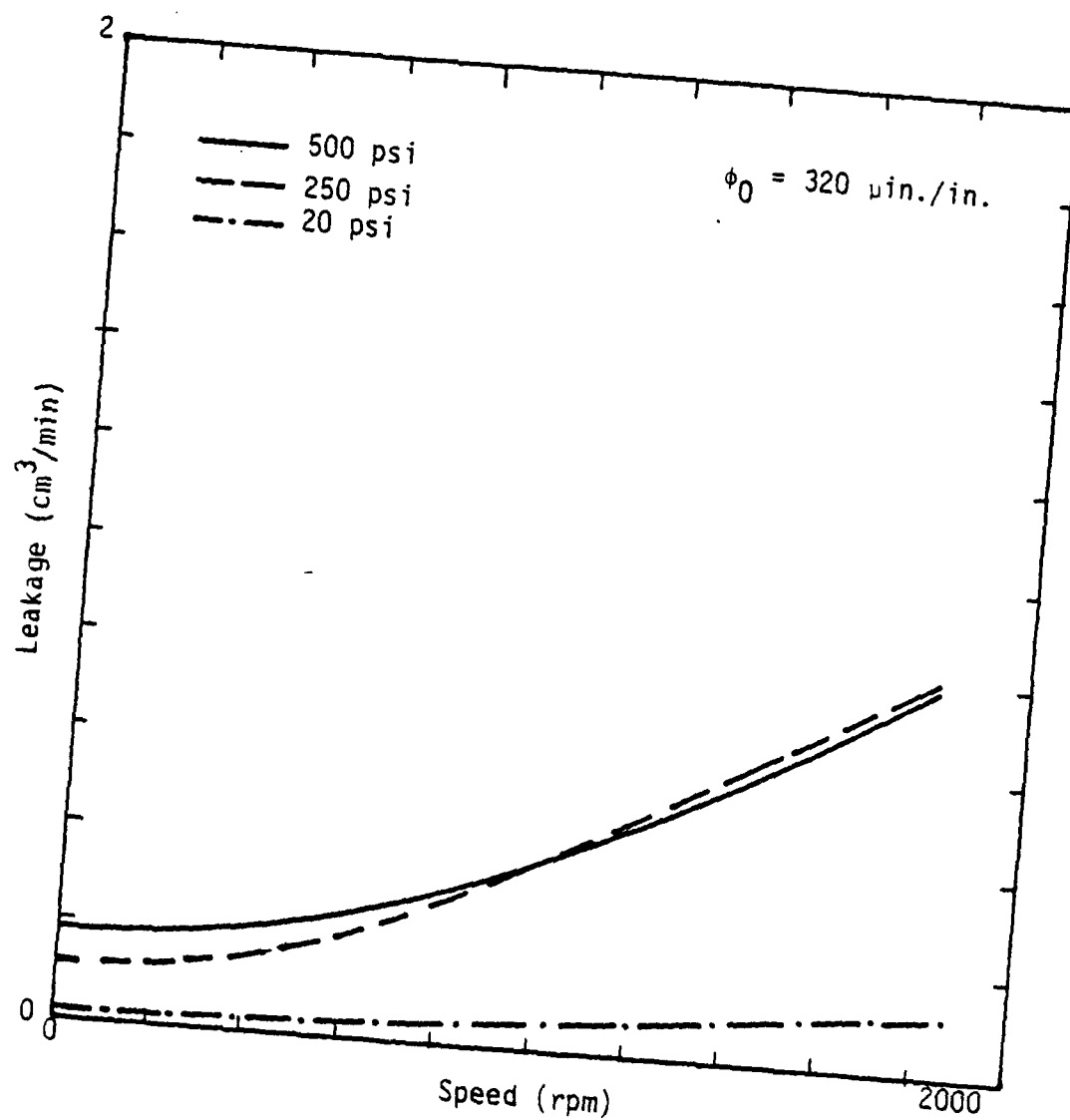


Figure 3-7. Predicted Performance - Leakage.



Thus, the  $\phi = 320$  was selected as a compromise between reduced friction and increased leakage.

Figure 3-8 shows a comparison between the worn in shape of the seal at two extreme operating conditions. Note the worn in shape is almost the same. These curves were calculated using another computer program which accounts for the deflections of the seal rings. Thus, as expected at nine waves, there is little change in the worn shape of the seal due to a change in operating conditions. Thus, no acceleration in wear would be expected in the design due to variable operating conditions. Furthermore, the additional small deflection component makes virtually no difference on the predicted performance.

#### Finite Element Calculation

To provide an additional check on the calculation for zero pressure rotation contained in the design program, a completely independent calculation for pressure caused rotation was made. Using SAP IV two-dimensional axisymmetrical elements, the cross section shown in Figure 3-5 was modeled using 161 nodes and 126 rectangular elements. The results showed that a rotation of 19  $\mu\text{in./in.}$  convergent resulted. This is quite small. This result shows that the beam theory used in the design program is adequate for calculating zero pressure moment designs and that the design itself is correct from the pressure rotation standpoint.

#### Strength and Deflection Calculations

Room temperature properties of annealed Inconel 625 are as follows:

$$S_u = 120 \text{ to } 150,000 \text{ psi}$$

$$S_y = 60 \text{ to } 95,000 \text{ psi} \quad (3-14)$$

$$S_e' = 90,000 \text{ psi at } 10^8 \text{ cycles}$$

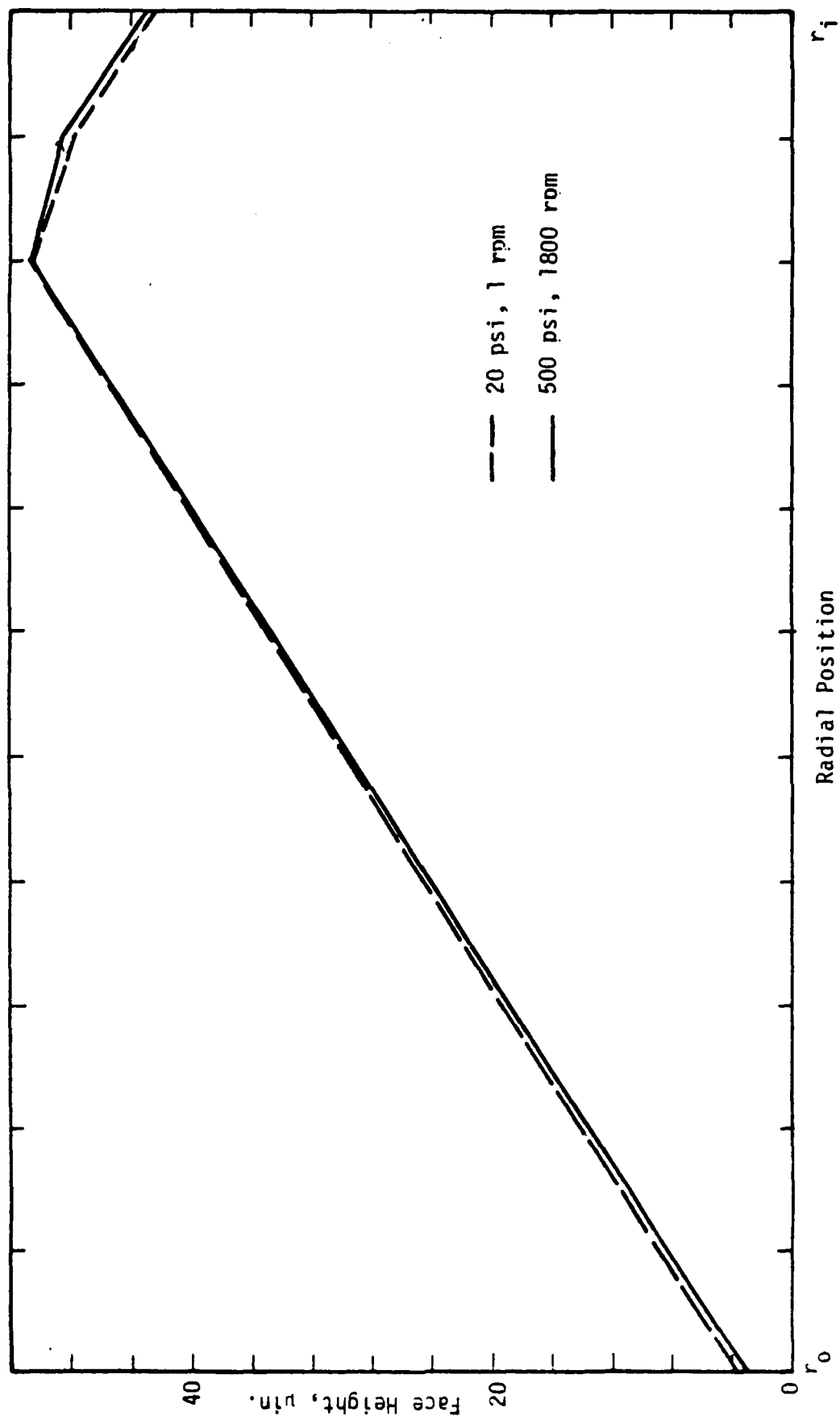


Figure 3-8. Predicted Worn Face Shape.

Several strength calculations were made. Points A and B shown in Figure 3-5 are highly stressed points.  $\sigma = 38,000$  was allowed at these points. This stress determined the pad thickness and the moment arm connection system, both of which are to be as small as possible from a space point of view. It was assumed that the moment arm itself does not deflect because of the deep cross section of the arm.

#### Other Calculations

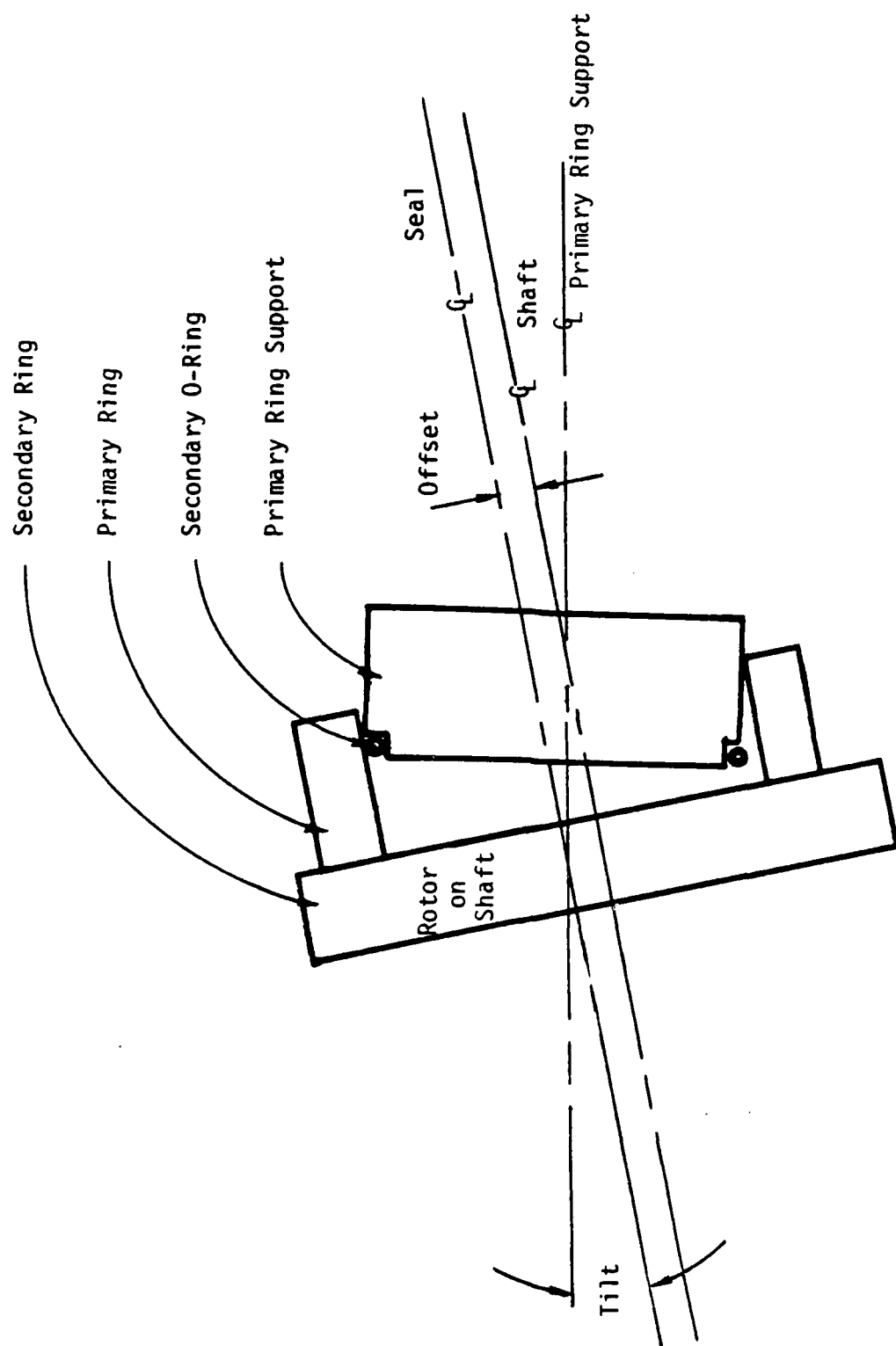
Many other design types of calculations were made. These include:

- 1) Ninth harmonic radial deflection (effects O-ring seals)
- 2) Assembly bolts (Monel and Inconel)
- 3) O-ring assembly
- 4) Spring pressure--spring design
- 5) Pressure drop in oil passages
- 6) Mating ring rotation
- 7) O-ring friction

#### Tilt and Offset

One of the test requirements for the simulated submarine operation test is to introduce known and variable tilt and offset misalignments in the seal. The purpose is to simulate the misalignment which results from hull deflection with variable pressure. In the test apparatus the tilt and misalignment are obtained by the rotation of a misaligned mounting cylinder.

Figure 3-9 shows the geometry of the required tilt and offset. Tilt and offset are in the same plane. The tilt and offset are specified at each simulated test depth. Figure 3-10 shows schematically how the tilt and offset are achieved in the test apparatus. The shaft is tilted and offset relative to the end-plate as shown. The mating ring axis coincides with the shaft



Notes: 1) Tilt and offset are in same plane.  
2) Housing (primary ring) is higher than shaft.

Figure 3-9. Required Offset and Tilt.

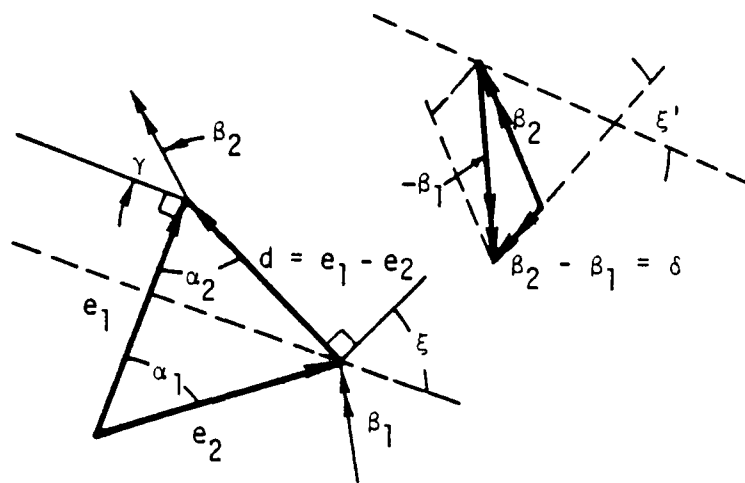
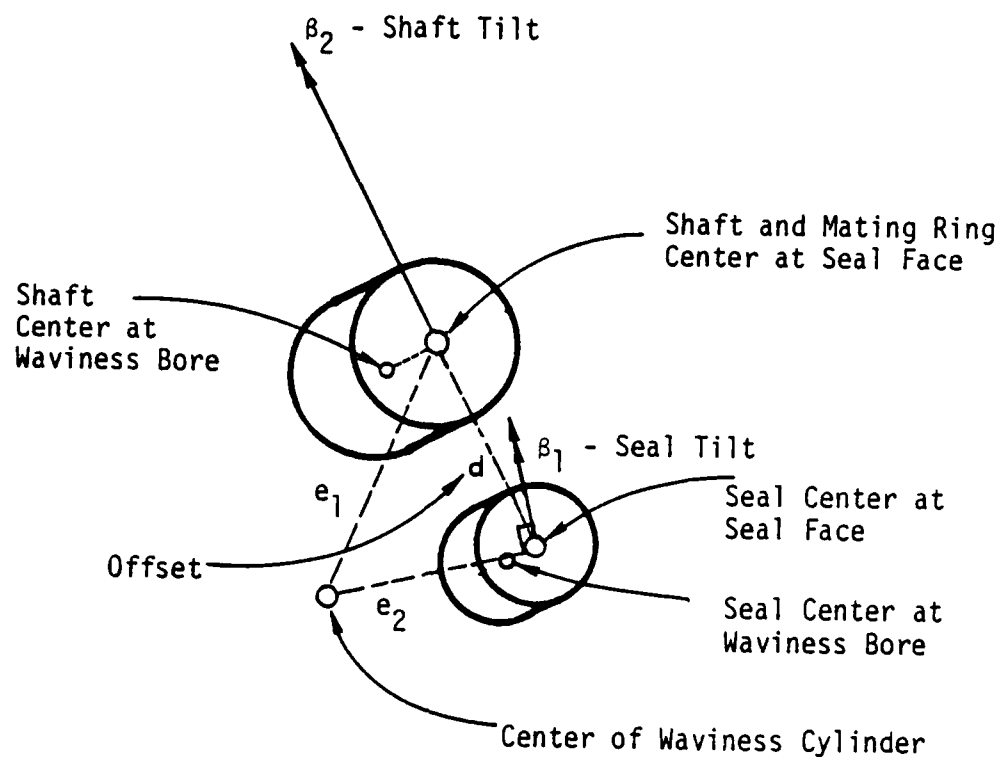


Figure 3-10. Method of Controlling Offset and Tilt.

axis. The seal axis is fixed to the waviness cylinder axis but it is in turn tilted and offset relative to the endplate. The relative offset between the shaft and the seal are of interest. Thus the relative offset is equal to the shaft offset vectorially minus the seal offset. The tilt is equal to the shaft tilt vectorially minus the seal tilt. The angle  $\alpha_1$  represents the positions of the waviness cylinder. If  $\alpha_1$  changes and all tilts and offsets are held fixed, their vector sums change with this rotation. Thus by varying the position of the waviness cylinder, it is possible to cause a variable offset and tilt. The problem of course, is to find all of the various tilt and offset angles which give the desired results.

Before the method of solution is presented, first the mathematics of the problem must be put forth. The parameters over which one has control either by manufacture or adjustment are:

- $\beta_1$  - seal tilt relative to endplate
- $\beta_2$  - shaft tilt relative to endplate
- $\gamma$  - angle of  $\beta_2$  relative to  $e_1$
- $e_1$  - shaft offset relative to endplate
- $e_2$  - seal offset relative to endplate
- $\alpha_1$  - angle of  $e_2$  relative to  $e_1$

Note  $\beta_1$  is taken as perpendicular to  $e_2$ . This is done to simply eliminate a variable. The needed offset and tilt are shown in Figure 3-10 as vector sums of the various components. Note that  $\xi$  and  $\xi'$  should be approximately equal so that the tilt and offset are in the same plane.

Now given the above values

$$d = \sqrt{e_1^2 + e_2^2 - 2e_1e_2 \cos \alpha_1}, \quad (3-15)$$

$$\alpha_2 = \cos^{-1} \left[ \frac{e_1^2 + d^2 - e_2^2}{2e_1 d} \right], \quad (3-16)$$

$$\xi = \alpha_2, \quad (3-17)$$

$$\delta = \sqrt{\beta_1^2 + \beta_2^2 - 2\beta_1\beta_2 \cos(\alpha_1 - \gamma)}, \quad (3-18)$$

$$\xi' = \cos^{-1} \left[ \frac{\delta^2 + \beta_2^2 - \beta_1^2}{2\delta\beta_2} \right] - \gamma. \quad (3-19)$$

Thus, the required parameter values may be calculated quite readily.

After some considerable trial and error, the following calculational method was devised. (Note from Table 3-2 that  $d$  and  $\delta$  are given as three sets of values which will be given subscripts  $a$ ,  $b$ , and  $c$ .)

- 1) Choose  $e_1$ ,  $e_2$ ,  $\beta_1$  and  $\beta_2$ .
- 2) Calculate  $\alpha_2$  from (3-16) and  $\alpha_1$  from the law of sines for all  $d$ .
- 3) Find  $d\delta/d\alpha_1 = (\delta_c - \delta_a)/(\alpha_{1c} - \alpha_{1a})$ .
- 4) Find  $\delta_b/(d\delta/d\alpha_1)$  from above.
- 5) Now, from (3-18)

$$\frac{\delta}{d\delta/d\alpha_1} = \frac{\beta_1^2 + \beta_2^2 - 2\beta_1\beta_2 \cos(\alpha_1 - \gamma)}{\beta_1\beta_2 \sin(\alpha_1 - \gamma)}. \quad (3-20)$$

Solve this for  $(\alpha_{1b} - \gamma)$ .

- 6) Find  $\gamma$  since  $\alpha_{1b}$  is known.
- 7) Calculate actual  $\delta_a \delta_b \delta_c$  and  $\xi$  and  $\xi'$ . Compare to desired results.

Table 3-2  
Offset and Tilt Results

Position	Desired Offset	Desired Tilt	Computed Offset $d$	Computed Tilt $\delta$	$\xi' - \xi$	$\alpha_1$
a	0.058 in.	0.0002	0.058 in.	0.00028	21°	56.2°
b	0.063 in.	0.0005	0.063 in.	0.00050	0°	61.6
c	0.067 in.	0.0007	0.067 in.	0.00070	8°	66.0

Computed results for:  $e_1 = 0.063$        $e_2 = 0.060$   
 $\beta_1 = 0.0029$        $\beta_2 = 0.0027$   
 $\gamma = 52.1^\circ$



specified by the test program, can be obtained. Details of the geometry related to this procedure are described in Chapter 3. Table 3-2 shows the final results for the selected parameter values shown. The offset values can be matched exactly. It was found that there was no way to exactly match the tilt values; so there is some error in  $\delta_a$ . It is also not possible to make  $\delta$  exactly perpendicular to  $d$  at all positions.  $\xi' - \xi$  shows the error in the first and last positions.

Given the reasons for introducing the tilt, it was judged that the above is sufficiently close to the desired to assess the effects of variable offset and tilt in operation. At the same time, the tilt and offset are readily changed by rotating the waviness cylinder. This can be performed by computer control and thus saves a great amount of repetitive manual adjusting.

#### Summary

The preceding sections provide brief descriptions for the computational portion of the nine-wave seal design. Details of the hardware itself are given in Chapter 2.

## CHAPTER 4

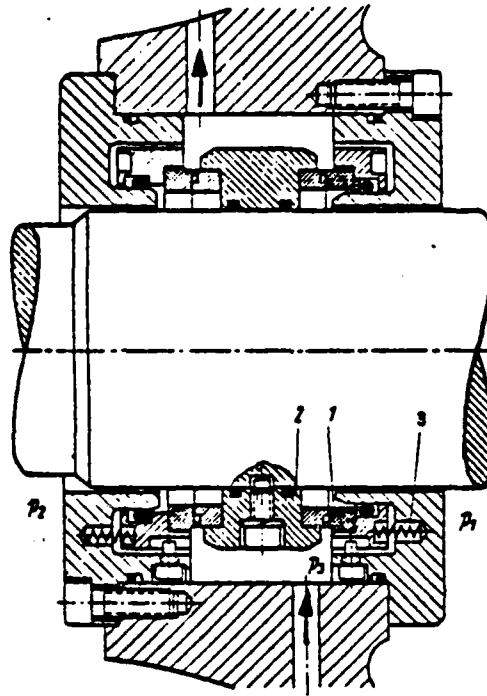
### MOVING WAVE GAS SEAL

#### Background

There are several types of gas seals being used in industry today, but due to the demands of advanced technology, a more reliable gas seal is still needed. Experimental work on the moving wave liquid seal has demonstrated some promising results in minimizing leakage, friction, and wear using this concept. The potential advantages of a moving wave gas seal are that 1) the seal will not foul by dirt or oil because the faces are continuously wiped by the moving wave action, 2) the seal will not be detrimentally affected by liquid contamination since it can be designed to operate in liquid and gas environments, 3) uneven wear caused by hot spots or contact can be smoothed out by the continual wearing in process of the seal, and 4) the seal is stiff and good tracking at high speeds would be expected. In this chapter, this concept has been applied to a gas seal. A moving wave gas seal has been modeled and the results are presented herein.

State-of-Art of the Gas Seals--It was not until the late 50s and early 60s that gas bearings and gas interface gas seals respectively were developed. In order to satisfy different kinds of applications and requirements, various types of gas seals have been tried. There are about four main types of gas seals.

A buffered gas seal is a double seal configuration (Figure 4-1) using oil as a buffer. The purpose of the buffer oil is to guard against the escape of the sealed fluid to the external atmosphere and also to provide lubrication between the seal faces. The buffered seal can seal corrosive gases up to 750 psi (5.17 MPa) with rubbing speeds up to 350 ft/s (106.7 m/s). The



1. Primary Ring
2. Secondary Ring
3. Spring

$P_1$  } Sealed Gas Pressures  
 $P_2$  }

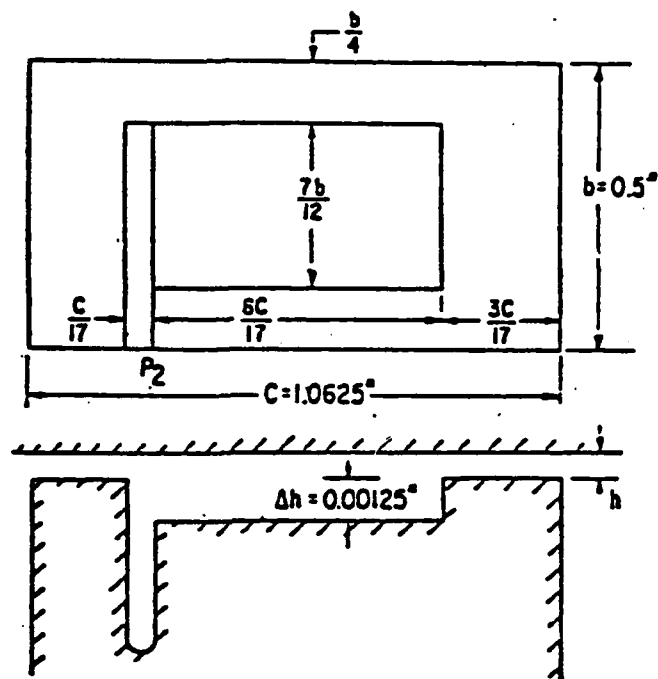
$P_3$  Buffered Oil Pressure

Figure 4-1. Buffered Gas Seal.

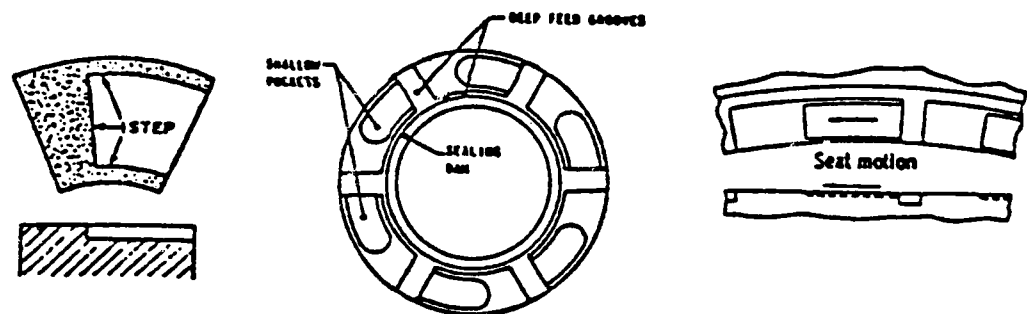
oil pressure is about 30 to 50 psi (0.21 to 0.34 MPa) higher than the system gas pressure. The buffer oil also acts as a coolant to the system. The major disadvantages of this gas seal are that a pressure pump, a filter, and a heat exchanger are required to deliver the buffer oil from an external source [25] and that the oil may contaminate the process fluid.

A shrouded Rayleigh step gas seal is shown in Figure 4-2. Basically, it contains a pumping land or dam and a pocket which is surrounded by a sealing area of small gap. There are other configurations as shown in Figure 4-2. When the secondary seal ring rotates, the Rayleigh pockets will scoop up the sealed gas and raise the gas pressure; at the same time the lands around the pocket will restrict the gas outflow. So the performance of this kind of seal depends on the land area to pocket area trade-off. This implies that if the land is wide, less area is available for pressure generation, but sealing is good. According to Cheng, Castelli and Chow [26], the leakage of this seal is largely determined by hydrodynamic action; the stronger the hydrodynamic action is, the more the leakage will be. Quantitatively speaking, the leakage rate is slightly higher than that of the spiral-groove orifice gas seal discussed later. The change of a pressure ratio ( $P_0/P_i$ ) has little effect upon the stiffness as well as the load support.

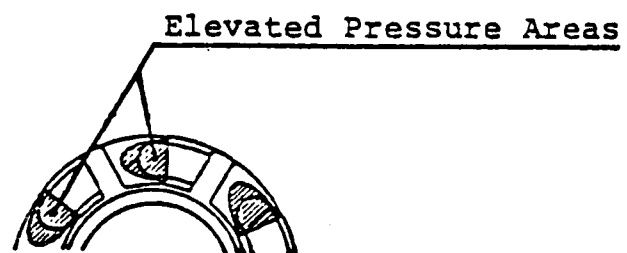
According to Zuk, Ludwig and Johnson [27], NASA has sponsored tests of the Rayleigh seal for future supersonic transport engines. The performance demonstrated the feasibility of operation at gas temperatures up to 1200°F (650°C), pressure differentials across the seal up to 250 psi (1.7 MPa) and relative surface speeds up to 450 ft/sec (140 m/sec). After 338.5 hours of testing, the average wear on the carbon seal was less than 5  $\mu$ in. (0.13  $\mu$ m). In the total 500 hours test, the seal had encountered over 50 startups and shutdowns and the leakage rates



(a) Typical Configuration of a Rayleigh Step Seal



(b) Other Configurations of Step Seals



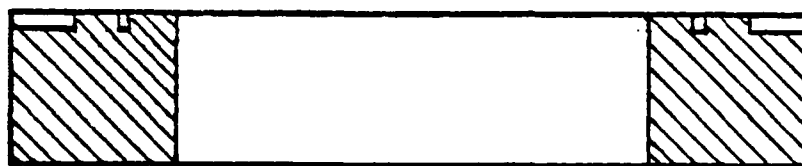
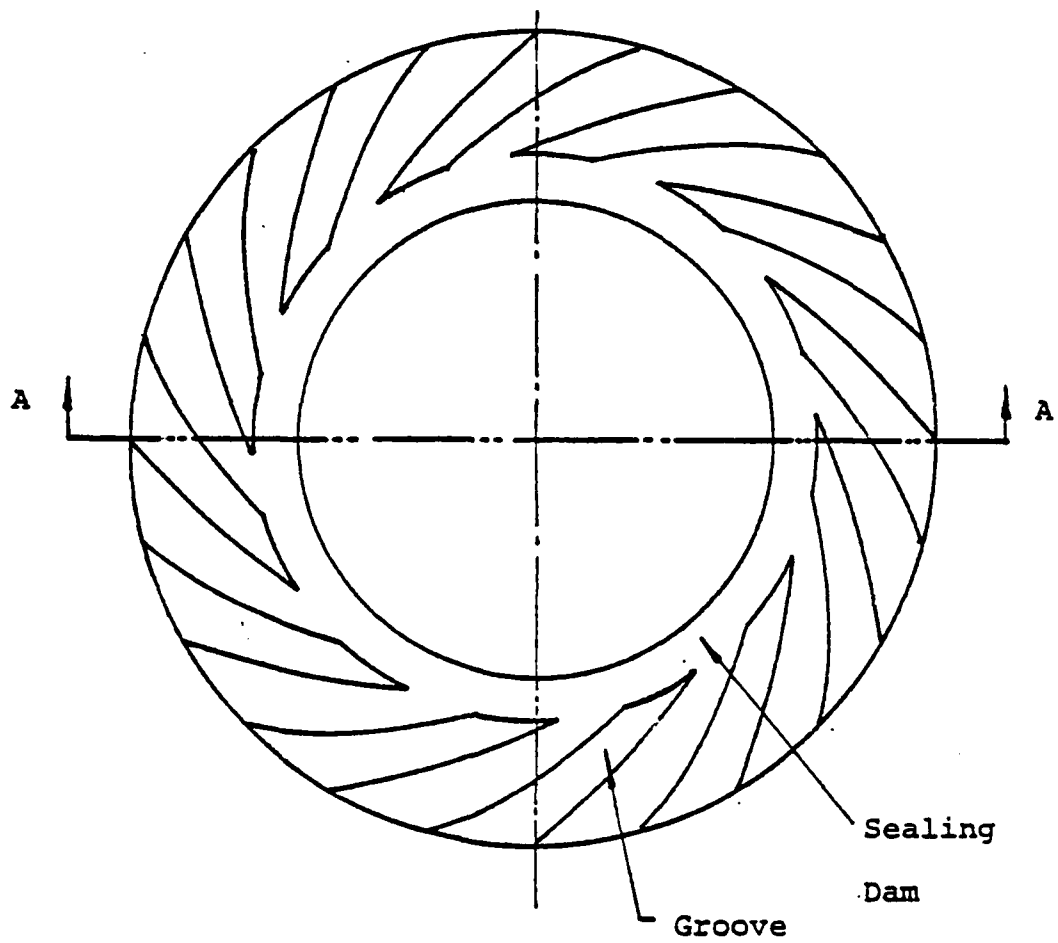
(c) High Pressure Areas

Figure 4-2. Shrouded Rayleigh Step.

varied from 11 to 32 SCFM (0.31 to 0.91 SCMM) with an average leakage about 25 SCFM (0.71 SCMM).

Among the various kind of gas seals, the so-called spiral-groove gas seal is well known. As shown in Figure 4-3, the surface of the secondary seal has been machined or etched in the form of spiral-grooves which are equally spaced. The spiral does not extend across the full width of the seal but terminates in a circumferential sealing dam. The purpose of the spiral is to pump the sealed gas inward while the dam restricts radial leakage. Thus, the gas is pressurized over the whole area of the seal and the seal load is supported by this pressure. According to Sedy [28], leakage rates as low as 0.1 SCFM at 850 psig (5.86 MPa) gas pressure on 4.5 inches (114.3 mm) diameter seal with 10,380 rpm speed have been recorded. Typically the leakage is about 1.9 SCFM (0.05 SCMM) in the same test and seal. The wear which occurred on the seal face during this test was about 4  $\mu$ in. (0.1  $\mu$ m) after 1000 hours.

At the present state-of-art, existing gas seals are satisfactory for many applications; leakage is relatively low and shaft speeds up to 20,000 rpm can be used for smaller seals. However, improvements are still needed. For instance, the gas film riding seals above are sensitive to the detrimental effects of dirt or liquid contamination. Dirt or liquid will alter the pressure distribution and abrasive wear may occur. Seal leakage rate and wear may dramatically change. High spots occur on most mechanical seal faces. As a result, hot spots may occur due to highly localized friction, and wear as well as driving torque may ultimately become higher. This behavior can lead to complete seal failure. Most gas film riding gas seals have relatively high leakage rates. With regard to these problems, further improvements in the state-of-art would be most beneficial. In pursuit of such improvements, the wavy gas seal deserves study.



Section A-A

Figure 4-3. Spiral Groove Gas Seal.

In terms of the required analysis the gas seal is somewhat different than the liquid seal studied previously herein. Thus it is useful to review the literature for studies of other types of gas seals to better understand what is important and the computational methods available.

Literature Review--In the late 1960s, many successful and efficient techniques of seal analysis were developed. First, James and Potter [29] solved for the pressure distribution for both spiral-groove thrust bearings and spiral-groove compressors. They derived a "jump" boundary equation between the land and the groove, and then made use of variable transformations to straighten out the groove for easier computation. To facilitate further the analysis, they changed the dependent variable from  $P$  to  $Q$  where  $Q = 1/2 P^2$ . Finally, they used an iterative technique to find the pressure distribution. Given the pressure distribution, the gas flow, load support, friction, static stiffness, power consumption and efficiency can be calculated.

Cheng, Castelli and Chow [26] made detailed analyses of two high speed noncontacting hybrid seals, the spiral-groove and Rayleigh step seals. The first part of their paper shows the performance of the spiral-groove seal with orifices and then without orifices. The differences in performance between the cases where the grooves are located at the high pressure and the low pressure or both sides were also studied.

In the Rayleigh step seal analysis [26], several assumptions were made: laminar and isothermal flow, Newtonian fluid, and negligible inertia. With the change of variable  $P^2 = Q$ , the Reynolds equation was expressed in finite difference form. The result is a set of equations with the pressure ( $Q$ ) at five points operated on by a linear algebraic operator. These equations were solved by the so-called matrix inversion technique developed from Reference [30]. This technique enables one to formulate the



$m \times n$  unknowns in  $m \times n$  linear equations with known coefficients. However, because pressure  $P$  acts as a variable coefficient (which is assumed to be a constant for each iteration), an iterative procedure was adopted to improve the guess values of  $P$ .

The second part of the Cheng, Castelli and Chow paper [26] gives typical performance for a non-parallel film profile spiral-groove-orifice seal as well as for the Rayleigh step seal. Finally, a comparison was made between the two seal geometries on their tolerance to tilt or coning under a constant seal load. The authors concluded that the difference in stiffness and leakage between the two seals is small, but the spiral-groove-orifice seal shows small variations in leakage rate as well as axial stiffness with respect to tilt or coning.

Gardener [31] has summarized and presented a numerical method for solving the design problem of a spiral groove noncontacting seal. Gardener considered pressure to be a function of radius only, so that the analysis was simplified. Some time later, Gardener [32] presented a paper covering additional developments on noncontacting spiral-groove seals. He pointed out several modifications to improve performance.

Zuk, Ludwig and Johnson [27] presented an approximate quasi-one-dimensional model for compressible fluid flow across face seals. Both the flat face and Rayleigh step gas seals were considered and tested. The model includes fluid inertia, viscous friction and entrance losses. It is valid for both laminar and turbulent flows. Both subsonic and choked flow conditions can be predicted by using an approximate integrated model (an iterative technique). Results agree with classical subsonic compressible viscous flow theory for Mach number less than  $1/\sqrt{\gamma}$ , where  $\gamma$  is the ratio of specific heats. If Mach number is equal to 1, sonic velocity and thus choked flow will occur. Results also show that near critical flow conditions (Mach number = 1), the film stiffness is negative, that is, as film thickness and Mach number

increase, the applied load actually increases. For choked flow the analysis predicts a high positive film thickness. The authors also found that as the sealing dam width increases, the critical pressure ratio (ambient pressure over sealed pressure) for choking decreases.

Later, Zuk [33] presented an application of the finite element method to compressible problems using a Galerkin solution technique. The most significant aspect of this paper is the inclusion of a fluid inertia term as well as choked flow conditions which make the compressible flow equations nonlinear. He also compares the advantages and disadvantages between the finite element method and the finite difference method.

In the late 1970s, Sedy [28] found that the performance of a spiral-groove gas seal can be improved through enhancement of hydrodynamic effects. Similar results are also applied to Rayleigh step gas seals. The methods of improving the performance include 1) using a high balance ratio, 2) widening the sealing dam, 3) increasing the overall face width, and 4) modifying the carbon sealing ring cross section.

Later, Sedy [34] presented a new self-aligning mechanism for improving the stability of spiral-groove gas seal. That is, when originally parallel faces of the mating rings deflect, the change in pressure profile tends to return the deflected ring toward its parallel position. It does not matter whether the deflection is convergent or divergent. If the gap is stable, the leakage and radial distortion are also stable. However, to benefit from this effect the pressure between the spiral-groove and the dam must be higher than the sealed pressure and the centroid location must be controlled. This mechanism not only minimizes angular deflection of the primary low-modulus seal ring, but also minimizes fluctuations in the gap width.

Concerning methods of solution for the pressure, from a mathematical point of view, gas seal analysis is more complicated

than liquid seal analysis. Although gas seal analysis is not a new branch of research, relevant information and techniques to solve for the pressure distribution are still being developed. In 1968, Castelli and Pirvics [35] reviewed different kinds of numerical methods for the solution of the gas bearing problems. The authors described some of the techniques using explicit, implicit and semi-implicit methods for the time-dependent Reynolds equation. The Alternating Direction Implicit method was also mentioned. Due to the nonlinear nature of the compressible Reynolds equation, solution techniques such as Gaussian Elimination become just a part of a larger iteration solution.

According to Fairweather and Mitchell [36], Alternating Direction Implicit (A.D.I.) methods have proven to be valuable finite difference methods for various types of partial differential equations, such as the parabolic and elliptic equations in two or three space variables. They discussed the computation procedure of the A.D.I. method from a pure numerical point of view. A paper by Evans and Gane [37] is a good example of the application of A.D.I. They were successful in applying this method to transient heat conduction problems. Also, Roache [38] has a concise description on the A.D.I. method. He generalized the experience of some researchers and concluded that A.D.I. methods do allow large time steps and fast overall computation.

One of the more useful references for bearing analysis was written by Shapiro [39]. He gives very detailed discussions of numerical techniques, presents several examples, and makes suggestions for future development. One of his examples is the A.D.I. scheme applied to a gas-lubricated bearing. He used a so-called "cell" approach which operates on the continuity equation rather than on Reynolds equation. This method is said to be more convenient in solving complex geometries than the direct Reynolds equation analysis.

Finally, according to Gargiulo and Gilmour [40], the A.D.I. method has been shown to be efficient, flexible and free from cumulative round off errors for the design of externally pressurized porous gas bearings. They used a systematic method to replace the Laplace difference operator in the pressure field by a finite difference approximation according to the A.D.I. scheme. They also gave a set of linear algebraic equations first in radial direction and then in the circumferential direction. The two tridiagonal systems were efficiently solved by an elimination algorithm.

### Theory

In this section, a model of a moving wave gas seal is presented. The moving wave gas seal can be described as a gas seal that has both moving waviness and tilt imposed upon the seal face of the primary seal ring (see Figure 1-3). This section includes the derivation of the difference equations from the Navier-Stokes equations and techniques for obtaining load support, leakage, and friction. Mesh effect, time step effect and convergence criteria are also investigated. Finally, a flow chart of the computer program and its subroutines and example solutions are presented.

Compressible Flow Equations--Compressible flow equations for a thin film are obtained from a simplification of the Navier-Stokes equations. The simplifying assumptions used are:

- 1) Gas viscosity is constant
- 2) Perfect gas
- 3) Body forces are negligible
- 4) Flow is laminar, there are no vortices or turbulence anywhere in the lubricant film
- 5) Inertial forces are negligible compared with the viscous forces
- 6) No slippage occurs at the sealing faces

7) The curvature of the fluid film surface is negligible

8) Compared with the two velocity gradients,  $\partial V_r / \partial z$  and  $\partial V_\theta / \partial z$ , all other velocity gradients are negligible.

Since these assumptions are widely accepted for gas seal analysis, further justification will not be given here. However, details are available in Reference [23].

One further assumption made is that the seal surface roughness as shown in Figure 4-4 does not significantly affect the resulting flow equations and that the mean film thickness may be used instead of the actual film thickness for computational purposes. This assumption is discussed in detail in Reference [4].

By applying the above assumptions, the Navier-Stokes equations in polar coordinates can be simplified to the following,

$$\frac{\partial P}{\partial r} = \eta \frac{\partial^2 V_r}{\partial z^2}, \quad (4-1)$$

$$\frac{\partial P}{r \partial \theta} = \eta \frac{\partial^2 V_\theta}{\partial z^2}, \quad (4-2)$$

$$\frac{\partial P}{\partial z} = 0. \quad (4-3)$$

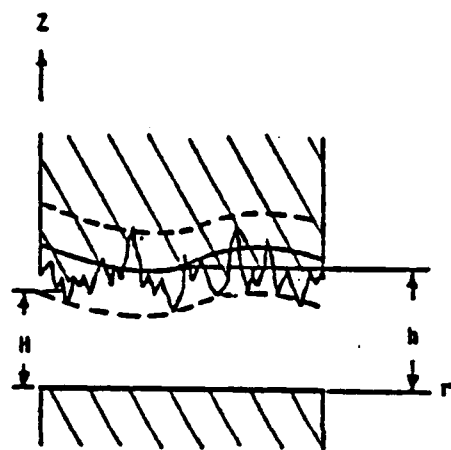
Figure 4-5 shows the applicable coordinate system. Integrating the above differential equations, the result is

$$V_r = \frac{1}{2\eta} \frac{\partial P}{\partial r} z^2 + C_1 z + C_2, \quad (4-4)$$

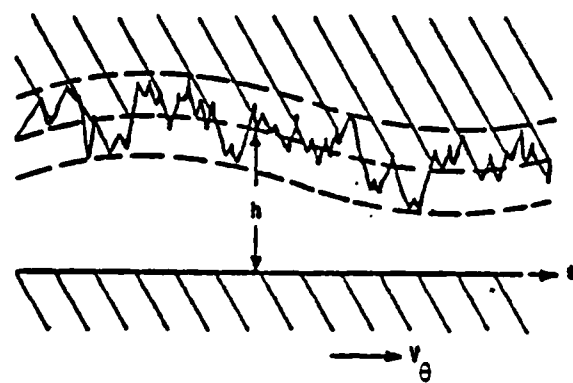
$$V_\theta = \frac{1}{2\eta r} \frac{\partial P}{\partial \theta} z^2 + C_3 z + C_4. \quad (4-5)$$

Applying the boundary conditions shown in Figure 4-5

$$z = 0 \quad V_{r1} = 0 \quad V_{\theta1} = \omega r \quad V_{z1} = 0, \quad (4-6)$$



Cross Section



Side View

Figure 4-4. Radial and Circumferential Profiles.

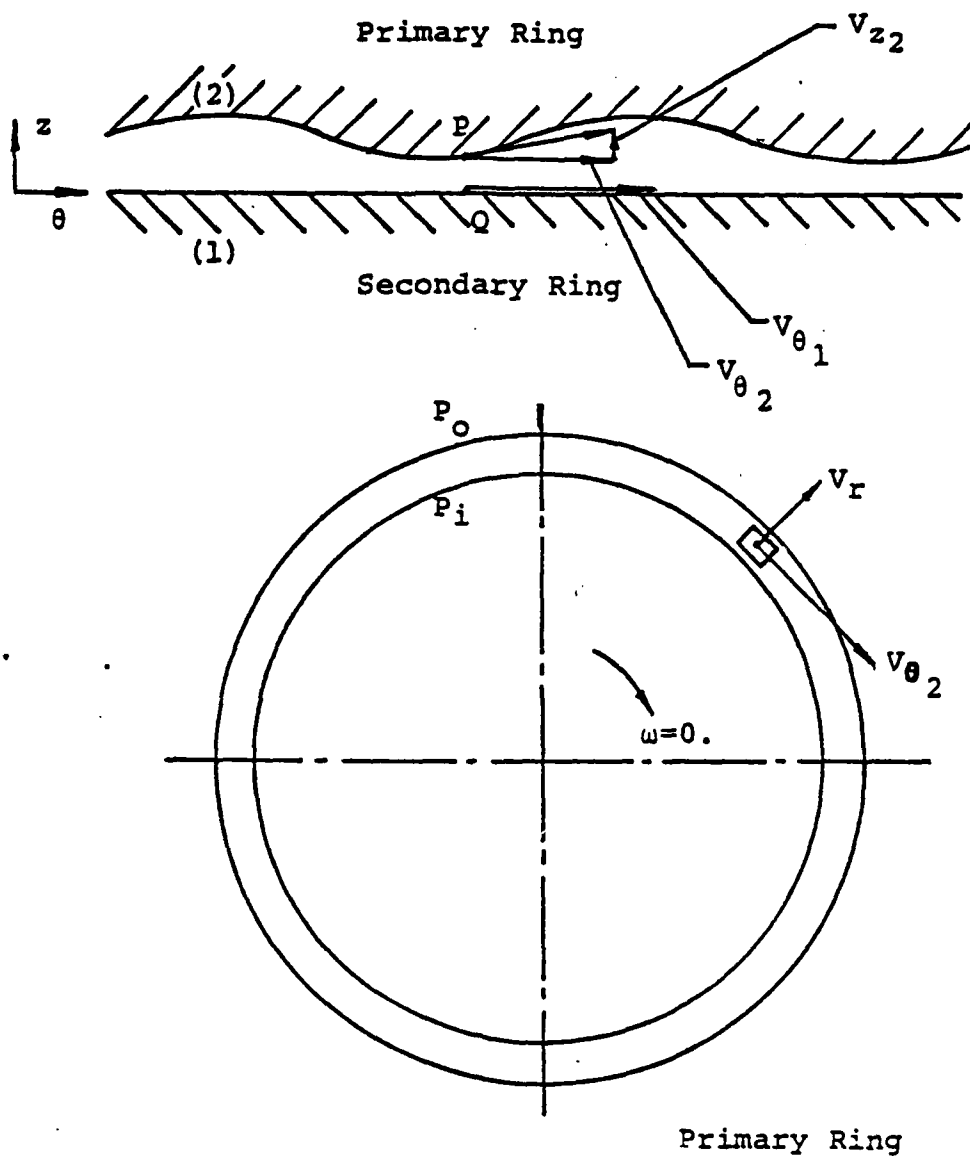


Figure 4-5. Boundary Conditions and Geometry.

$$z = h \quad v_{r_2} = 0 \quad v_{\theta_2} = 0 \quad v_{z_2} = 0 , \quad (4-7)$$

we then obtain two component equations of the fluid flow,

$$v_r = \frac{\partial P}{2\eta \partial r} (z^2 - zh) , \quad (4-8)$$

$$v_\theta = \frac{\partial P}{2r\eta \partial \theta} (z^2 - zh) + \omega r(1 - \frac{z}{h}) . \quad (4-9)$$

The mass flow rate is,

$$m_r = \int_0^h \rho v_r (r d\theta) dz , \quad (4-10)$$

$$m_r = - \frac{\rho r \Delta \theta}{12\eta} h^3 \frac{\partial P}{\partial r} , \quad (4-11)$$

$$m_\theta = - \frac{\rho \Delta r}{2} \left[ \frac{h^3}{6\eta r} \frac{\partial P}{\partial \theta} - \omega r h \right] . \quad (4-12)$$

The continuity equations for an element of volume is

$$\frac{\partial}{\partial t} \int_V \rho dV + \int_S \rho \vec{V} \cdot \vec{n} ds = 0 . \quad (4-13)$$

With Reference to Figure (4-6) the continuity equation can be written as,

$$\frac{\partial}{\partial t} \int_0^h \rho \Delta a dz + m_{j+1/2} + m_{i+1/2} + m_{j-1/2} + m_{i-1/2} = 0 , \quad (4-14)$$

where the 1/2 points refer to mass flow halfway between nodes. The mass change inside the control volume can be written in differential form as



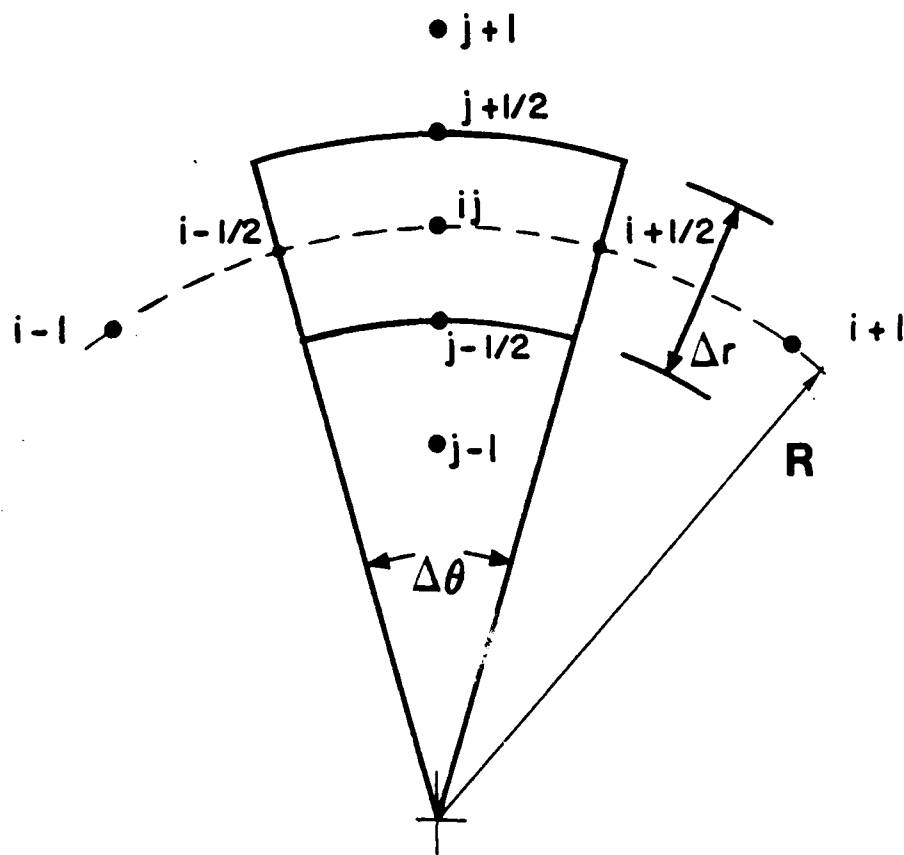


Figure 4-6. Control Volume.

$$m_V = \frac{\partial}{\partial t} [\rho h \Delta a] , \quad (4-15)$$

where  $\Delta a$  = surface area of top or bottom of the control volume.

Using Equations (4-11) and (4-12) the mass flows are

$$m_{j+1/2} = - \left[ \frac{r \Delta \theta \rho h^3}{12 \eta} \frac{\partial P}{\partial r} \right]_{j+1/2} , \quad (4-16)$$

$$m_{j-1/2} = + \left[ \frac{r \Delta \theta \rho h^3}{12 \eta} \frac{\partial P}{\partial r} \right]_{j-1/2} , \quad (4-17)$$

$$m_{i+1/2} = - \left[ \Delta r \frac{\rho h^3}{12 \eta r} \frac{\partial P}{\partial \theta} - \frac{\rho \omega r h}{2} \right]_{i+1/2} , \quad (4-18)$$

$$m_{i-1/2} = + \left[ \Delta r \frac{\rho h^3}{12 \eta r} \frac{\partial P}{\partial \theta} - \frac{\rho \omega r h}{2} \right]_{i-1/2} . \quad (4-19)$$

Substituting Equations (4-15) through (4-19) into Equation (4-14), assuming a perfect gas, isothermal conditions and  $\partial h / \partial t \rightarrow 0$  (no squeeze), one gets

$$\begin{aligned} & \left[ 12 \eta h \frac{\partial P}{\partial t} \right]_{i,j} - \left[ \frac{r \Delta \theta \rho h^3}{\Delta a} \frac{\partial P}{\partial r} \right]_{j+1/2} + \left[ \frac{r \Delta \theta \rho h^3}{\Delta a} \frac{\partial P}{\partial r} \right]_{j-1/2} \\ & + \left[ \frac{\Delta r \rho h^3}{r \Delta a} \frac{\partial P}{\partial \theta} \right]_{i-1/2} - \left[ \frac{\Delta r \rho h^3}{r \Delta a} \frac{\partial P}{\partial \theta} \right]_{i+1/2} \\ & + \frac{6 \eta \Delta r \omega}{\Delta a} [(Prh)_{i+1/2} - (Prh)_{i-1/2}] = 0 . \end{aligned} \quad (4-20)$$

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APPLICATION OF THE WAVY MECHANICAL FACE SEAL TO  
SUBMARINE SEAL DESIGN(U) NEW MEXICO UNIV ALBUQUERQUE  
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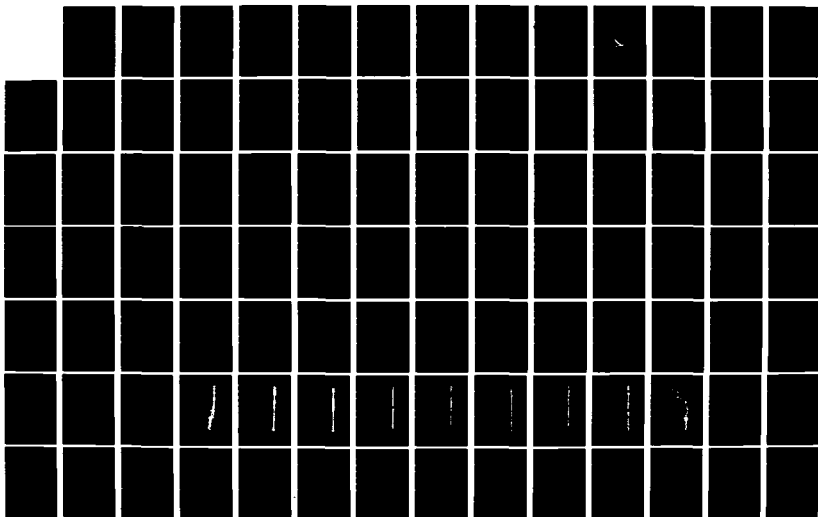
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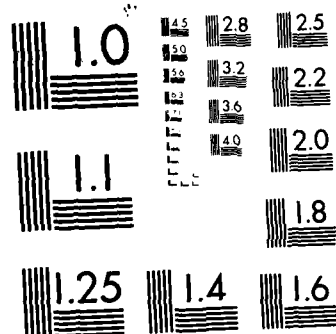
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To linearize the problem a change of variable is made,

$$Q = p^2 \quad (4-21)$$

so that

$$p \frac{\partial p}{\partial \theta} = \frac{1}{2} \frac{\partial Q}{\partial \theta}, \text{ etc.} \quad (4-22)$$

Incorporating this change in Equation (4-20) and using central differences to approximate the derivatives,

$$\begin{aligned} & \frac{12nh_{i,j}}{p_{i,j}^*} \frac{(Q_{i,j} - \bar{Q}_{i,j})}{2\Delta t} - \frac{(h^3 r)_{j+1/2} \Delta \theta}{2\Delta a} \frac{(Q_{i,j+1} - Q_{i,j})}{\Delta r} \\ & + \frac{(h^3 r)_{j-1/2} \Delta \theta}{2\Delta a} \frac{(Q_{i,j} - Q_{i,j-1})}{\Delta r} + \frac{h_{i-1/2}^3 \Delta r}{2r_{i,j}} \frac{(Q_{i,j} - Q_{i-1,j})}{\Delta a \Delta \theta} \\ & - \frac{h_{i+1/2}^3 \Delta r}{2r_{i,j}} \frac{(Q_{i+1,j} - Q_{i,j})}{\Delta a \Delta \theta} + \frac{6nwr_{i,j} \Delta r}{2\Delta a} \\ & \times \left\{ \left[ \frac{Q_{i+1,j}}{p_{i+1,j}^*} + \frac{Q_{i,j}}{p_{i,j}^*} \right] h_{i+1/2} - \left[ \frac{Q_{i,j}}{p_{i,j}^*} + \frac{Q_{i-1,j}}{p_{i-1,j}^*} \right] h_{i-1/2} \right\} \\ & = 0. \end{aligned} \quad (4-23)$$

where  $\bar{Q}$  is the value of  $Q$  at a previous time step and the remaining  $P^*$ 's are assumed known from a previous iteration. Since  $\bar{Q}$  and the remaining  $P$ 's are assumed known, Equations (4-23) become linear equations at any time step with variable  $Q$ . Grouping all the like terms as coefficients, the above equation becomes,

$$\begin{aligned}
& Q_{i,j} \left[ \frac{12h_{i,j} \Delta a}{P_{i,j}^* \Delta t \Delta r} + \frac{(h_{i,j+1/2})^3 r_{i,j+1/2} \Delta \theta}{\Delta r \Delta r} \right. \\
& \quad \left. - \frac{(h_{i,j+1/2})^3 r_{i,j-1/2} \Delta \theta}{\Delta r \Delta r} \right] \\
& + Q_{i,j+1} \left[ - \frac{(h_{i,j+1/2})^3 r_{i,j+1/2} \Delta \theta}{\Delta r \Delta r} \right] \\
& + Q_{i,j-1} \left[ - \frac{(h_{i,j-1/2})^3 r_{i,j-1/2} \Delta \theta}{\Delta r \Delta r} \right] \\
& = \bar{Q}_{i,j} \left[ \frac{12\eta \Delta a h_{i,j}}{P_{i,j}^* \Delta t \Delta r} - \frac{(h_{i-1/2,j})^3}{r_{i,j} \Delta \theta} - \frac{(h_{i+1/2,j})^3}{r_{i,j} \Delta \theta} \right. \\
& \quad \left. - \frac{\Delta r_{i,j}}{P_{i,j}^*} (h_{i+1/2,j} - h_{i-1/2,j}) \right] \\
& + \bar{Q}_{i+1,j} \left[ \frac{(h_{i+1/2,j})^3}{r_{i,j} \Delta \theta} + \frac{r_{i,j} h_{i+1/2,j}}{P_{i+1,j}^*} \right] \\
& + \bar{Q}_{i-1,j} \left[ \frac{(h_{i-1/2,j})^3}{r_{i,j} \Delta \theta} + \frac{r_{i,j} h_{i-1/2,j}}{P_{i-1,j}^*} \right]. \quad (4-27)
\end{aligned}$$

A special Gaussian elimination subroutine is used to solve the above tridiagonal equations. It stores the diagonal coefficients in a compact column matrix and takes advantage of the sparseness of the matrix to save computer time.

Boundary Conditions--The boundary conditions for the gas seal are similar to those for a liquid seal. That is,

$$P(r_i) = P_i ,$$

$$P(r_o) = P_o , \quad (4-28)$$

where  $P_i$  and  $P_o$  are the inside and outside pressures of the seal, respectively. Within the seal region pressures are set initially at time zero to values based on linear interpolations between the inside and outside pressure.

**Mesh effect**--One measure of solution accuracy is to compare the leakage rates at the inside and outside radii of the seal. Figure 4-7 shows this comparison as a function of tilt for two mesh sizes. A different grid size results in different leakage rates, but the difference is decreased by a finer mesh. The error increases with increasing tilt because a greater change in gap across the seal is produced.

Presumably, the leakages will converge to each other at all tilt angles if one can refine the mesh as small as possible. In practice one cannot refine the mesh too far because of cost. Hence, grid size comparison is terminated at the 19 x 20 mesh. All the results obtained are based on this mesh scheme.

**Convergence and Time Step**--Using the ADI method, one solves for the pressure field as a function of time. In this steady-state problem, one is actually solving for the transient response from an arbitrary starting pressure field to that which is the steady-state solution keeping the film thickness constant with time. A measure of satisfactory convergence is the total load supported by the pressure. Presumably, when this load stops changing with time, steady state has been reached.

In order to check the stability and convergence of the program, a series of data was computed and is plotted in

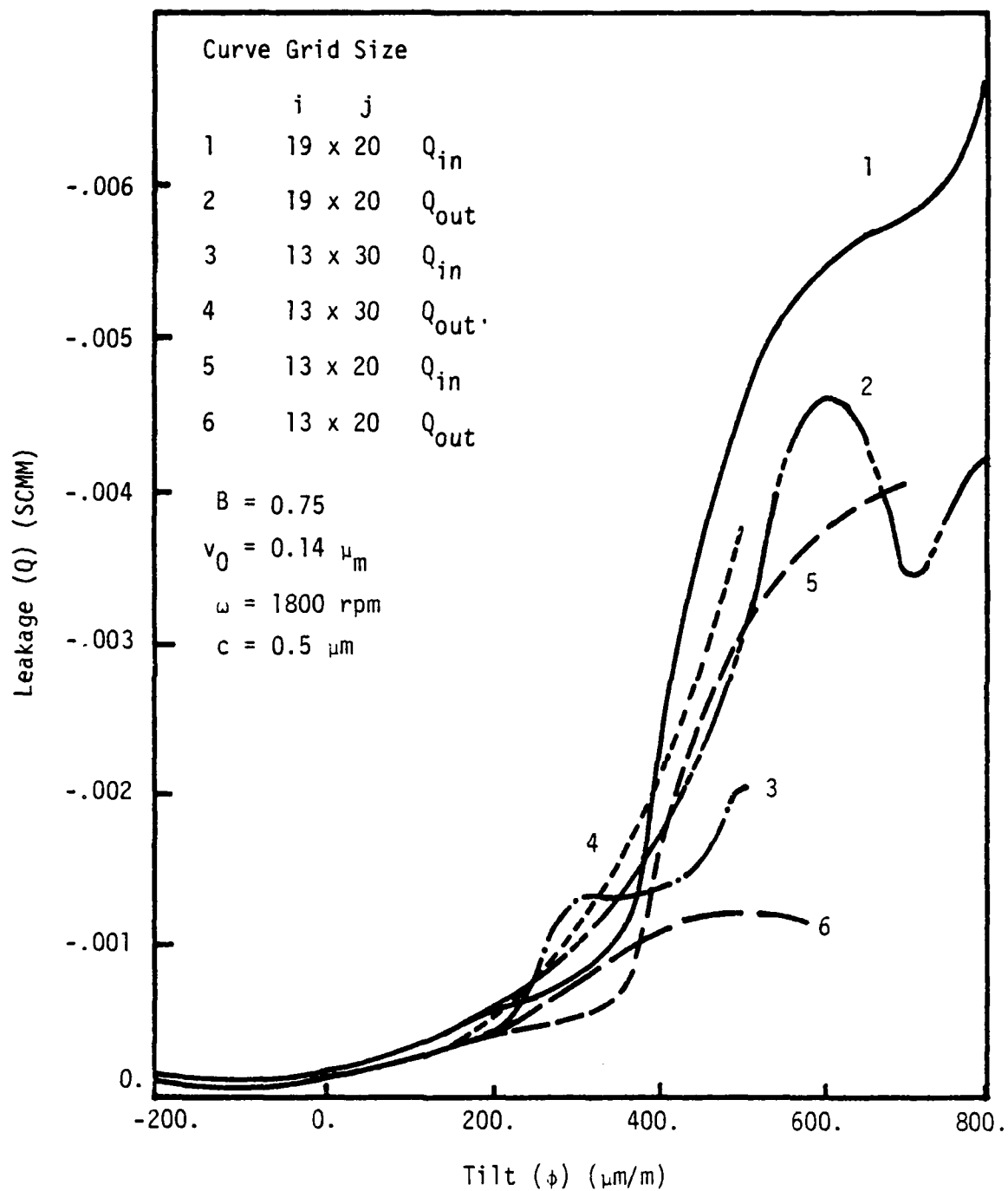


Figure 4-7. Leakage as a Function of Tilt.



Figure 4-8. In this figure, the maximum difference in absolute pressure ( $\Delta P$ ) between the old iteration values and the new values is used as a convergence criterion. The effect of pressure error and the time step on the fluid load support and the total computation time is shown. The figures show clearly that the pressure error can be used to get a satisfactory convergence on load support.

It is shown that a larger pressure error criterion requires a larger time step to obtain proper convergence on load. Less computational time is taken using this approach. If the pressure error remains the same while the time step is decreased as shown in Figure 4-8, a false or premature convergence results. On the other hand, if the pressure error is decreased, very consistent load support is found at various time steps. However, if the pressure error is decreased even more (e.g.,  $\Delta P < 5.5$  Pa), the execution for the larger time step ( $\Delta T = 5 \times 10^{-5}$  S) will be terminated because negative pressures occur. Also, the program loops without stopping for the smallest time step ( $\Delta T = 5 \times 10^{-7}$  S) because the pressure error ( $\Delta P$ ) is too small to meet.

As shown, care must be taken in order to pick a suitable pressure error and time step. Otherwise, too much computer time or false convergence will result. In this investigation, the pressure error criterion is taken to be the sum of the absolute changes for all the nodes. This value is taken at 3.5 KPa for 18 x 18 nodes. This corresponds approximately to 11 Pa in Figure 4-8. The time step is chosen to be  $5 \times 10^{-6}$  seconds.

Seal System--the moving wave gas seal can be defined by two sets of parameters, application parameters and design parameters. The former category includes the pressure differential to be sealed ( $P_o - P_i$ ), the angular speed ( $\omega$ ) of the shaft, shaft diameter ( $D$ ), seal centroid radius ( $r_c$ ) and fluid viscosity ( $\eta$ ). All of these parameters are fixed because of the requirements of a particular seal application.

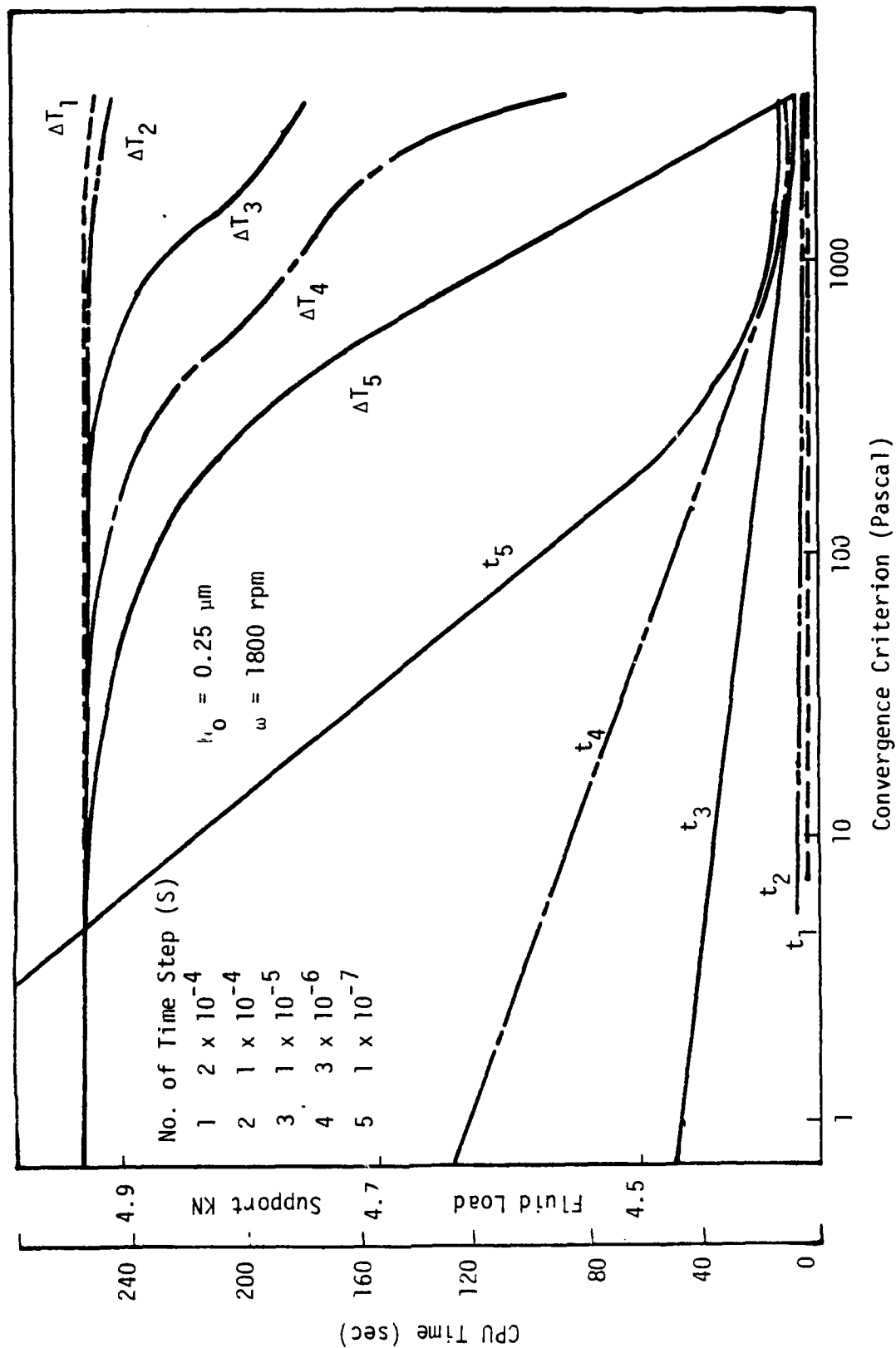


Figure 4-8. Convergence Conditions.

The design parameters consist of balance ratio (B), material properties such as compressive strength ( $P_m$ ), seal surface roughness (c), seal face width ( $r_o - r_i$ ), number of waves around the seal face (n), tilt angle ( $\phi$ ), amplitude of the waves, the ring stiffness ratio (A), spring pressure ( $P_{sp}$ ) and modulus of elasticity of the primary seal ring (E). The values of the above application parameters, design parameters as well as some constants are given in Table 4-1. Most of the system parameters are shown in the previous figures, such as Figure 1-3, Figure 4-4, and Figure 4-5. The definitions used are the same as those in previous reports.

Film Thickness Shape--the nature of the wavy tilted shape of the gas film in the seal is described in detail in Reference [5]. Film thickness is essentially a function of wear, deflection, and initial shape. For a moving wave or tilted seal, the film thickness function has been shown to be [5]:

$$h = h_o + w(r) + [v_o + (r - r_c) \phi_o] \cos(n\theta + \Omega t) \quad (4-29)$$

where

$$w(r) = -\min\{[v_o + (r - r_c) \phi_o] \cos(n\theta + \Omega t)\} \quad (4-30)$$

$v_o$  and  $\phi_o$  can be somewhat arbitrarily chosen, and their choice influences the shape of the film. In this gas seal study,  $v_o$  and  $\phi_o$  are chosen just as in the liquid seal work [5] such that there is a circumferential wave, a radial taper, and a distinct sealing dam.  $\Omega$  represents the wave motion required to maintain the wave in spite of wear and is so small that it can be neglected in the pressure computations. Figure 4-9 shows a three-dimensional plot of the wavy film thickness for a particular  $\phi$  and  $v$ .

Table 4-1

## Gas Seal Parameters and Constants

Face Seal Application Parameters

Viscosity of nitrogen	$\eta - 1.74 \times 10^{-5} \text{ Pa}\cdot\text{S} (2.52 \times 10^{-9} \text{ lbf}\cdot\text{s/in.}^2)$
Outside pressure	$p_o - 3.55 \text{ MPa} (514.7 \text{ psia}) (\text{varies})$
Inside pressure	$p_i - 0.1 \text{ MPa} (14.7 \text{ psia})$
Angular speed of shaft	$\omega - 1800 \text{ rpm} (188 \text{ rps}) (\text{varies})$
Seal centroid radius	$r_c - 49.18 \text{ mm} (1.9361 \text{ in.})$
Shaft diameter	$D - 88.90 \text{ mm} (3.50 \text{ in.})$

Face Seal Design Parameters

Ring Stiffness Ratio	$A - EJ_x/GJ_\theta = 7.44$
Compressive strength of carbon ring	$p_m - 262 \text{ MPa} (38,000 \text{ psia})$
Balance ratio	$B - 0.75$
Surface roughness	$c - 0.51 \text{ }\mu\text{m} (20.0 \text{ }\mu\text{in.}) (\text{varies})$
Face width: inside radius	$r_i - 48.26 \text{ mm} (1.9000 \text{ in.})$
outside radius	$r_o - 53.04 \text{ mm} (2.0875 \text{ in.})$
Number of waves	$n - 9$
Waviness amplitude	$h_o - 0.14 \text{ }\mu\text{m} (5.4 \text{ }\mu\text{in.})$
Spring pressure	$P_{sp} - 0.196 \text{ MPa} (28.5 \text{ psi})$
Modulus of elasticity	$E - 20.7 \text{ GPa} (3.0 \times 10^6 \text{ psi})$
Shear modulus	$G - 9.6 \text{ GPa} (1.25 \times 10^6 \text{ psi})$
Tilt	$\phi - 500 \text{ }\mu\text{m/m} (\text{varies})$

Other Constants

Gravity	$g - 9.81 \text{ m/s}^2 (32.174 \text{ ft/s}^2)$
Density of nitrogen	$\rho - 1.18 \text{ Kg/m}^3 (0.0023 \text{ slug/ft}^3)$
Spring constant	$k - 3.36 \text{ KN/m} (19.159 \text{ lb/in.})$
Temperature	$T - 23.9^\circ\text{C} (75.0^\circ\text{F})$
Universal constant	$U - 287 \text{ J/Kg}^\circ\text{K} (53.3 \text{ ft}\cdot\text{lb/lbm/R}^\circ)$

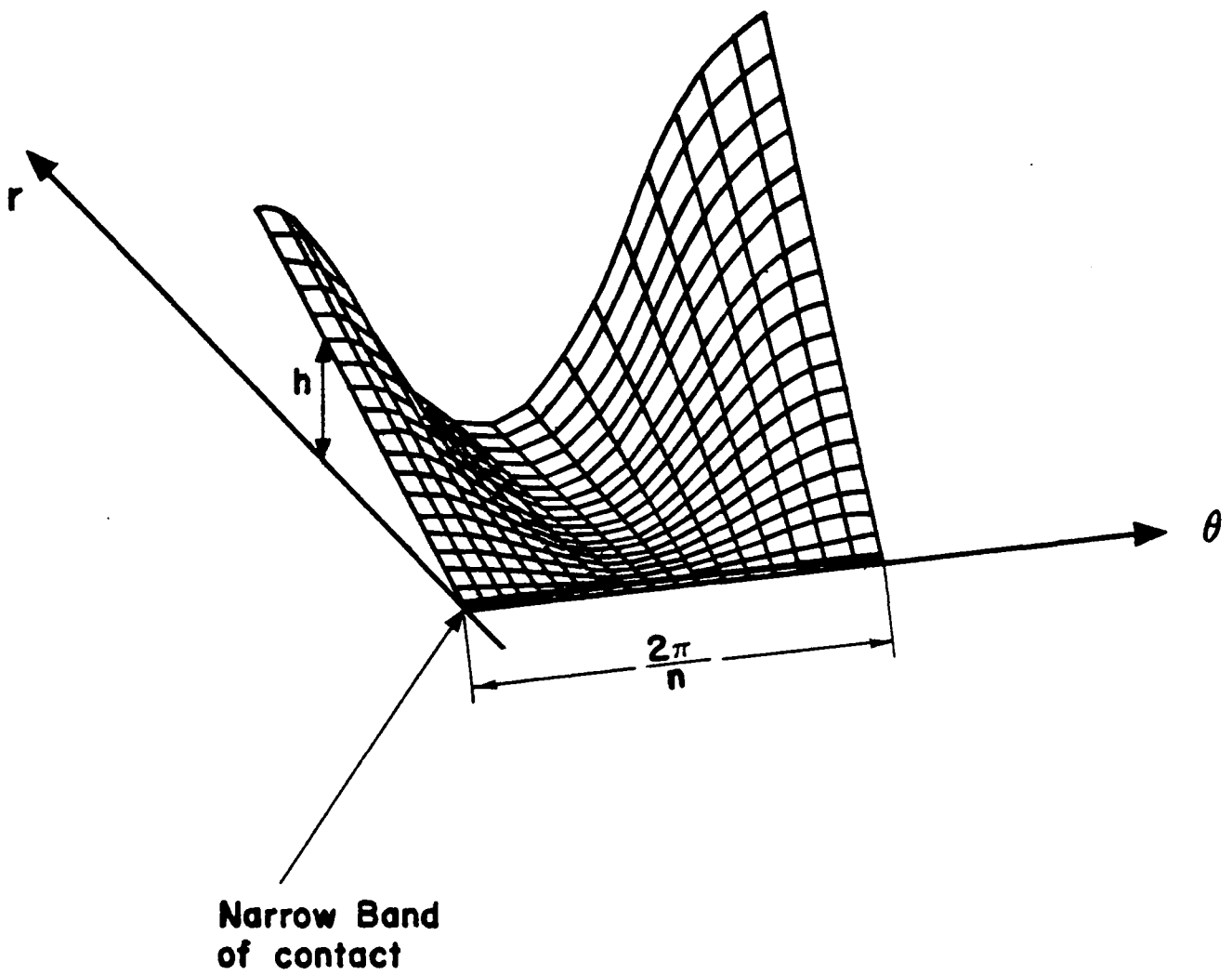


Figure 4-9. 3-D Plot of Film Thickness.

Load Support--Load support consists of the fluid pressure component and the mechanical component. The fluid pressure component is due to the hydrostatic and hydrodynamic effects while the mechanical component is obtained by flattening the peaks of the asperities in contact. As before, it is assumed that asperity pressure is equal to the compressive strength of the softer seal face material, and that this pressure remains constant.

The total load support in the gas seal is calculated by integrating the hydrodynamic, hydrostatic and asperity pressure distributions over the entire seal face area. That is

$$W = n \int_0^{\frac{2\pi}{n}} \int_{r_i}^{r_o} [P + P_m(1 - b)] r dr d\theta, \quad (4-31)$$

where  $b$  is the fraction of the face not in mechanical contact (given in Reference [6]).

The load that must be supported for the seal to be in equilibrium is a function of the seal pressures  $P_o$  and  $P_i$ , the balance ratio  $B$ , and the spring pressures  $P_{sp}$ .

$$W^* = \pi(r_o^2 - r_i^2) [P_o B + P_i(1 - B) + P_{sp}] \quad (4-32)$$

As before [1-5], the total load support  $W$  increases with decreasing  $h_o$ , thus, this load support may be adjusted to be equal to the applied load  $W^*$  by selecting a suitable  $h_o$ . Such a procedure is used for the solutions presented herein.

For comparison purposes, the ratio of fluid pressure load support to the total load ( $W_f/W$ ) expressed as a percent is an index to indicate the relative wear rate. If the ratio is equal to one ( $W_f/W = 1.0$ ) theoretically, adhesive wear will become zero.

Leakage--Gas seal leakage is computed by the integration of the equations

$$Q_{out} = - \int_0^{2\pi} \frac{h_o^3}{12\eta} \frac{\partial P}{\partial r} \bigg|_{r_o} r_o d\theta \quad (4-33)$$

and

$$Q_{in} = - \int_0^{2\pi} \frac{h_i^3}{12\eta} \frac{\partial P}{\partial r} \bigg|_{r_i} r_i d\theta , \quad (4-34)$$

$Q_{in}$  and  $Q_{out}$  represent the inside and outside leakage, respectively. Both the inside and outside leakages are used as a check to see whether accuracy is acceptable. A numerical average of the inside and outside leakages is used to represent the nominal seal leakage.

Friction--From the previous work [6], considering the fluid film regime, tangential fluid friction shear stress is given by

$$\tau_f = \left[ \eta \omega r E\left(\frac{1}{H}\right) + \frac{1}{2r} \frac{\partial P}{\partial \theta} E(H) \right] \times b \quad (4-35)$$

where  $E( )$  is the expectancy operator and  $b$  is the fraction of the face not in contact and has been given previously [6].

Roughness is taken as being given by a parabolic distribution as before,

$$f(h_S) = \frac{35}{32c} (c^2 - h_S^2)^3 , \quad -c < h < c . \quad (4-36)$$

The expected value functions are given in Reference [6].

The average friction shear stress due to asperity contact is given by,

$$\tau_a = P_S(1 - b) . \quad (4-37)$$

The mechanical torque is calculated by the integration of the tangential fluid shear stress plus the average frictional shear stress due to the asperity contact over the whole seal face. Thus,

$$\text{Torq} = n \int_0^{\frac{2\pi}{n}} \int_{r_i}^{r_o} (\tau_f + \tau_a) r^2 dr d\theta \quad (4-38)$$

and the mechanical driving power required is,

$$\text{Power} = \omega \times \text{Torq} . \quad (4-39)$$

The coefficient of friction is defined as

$$\mu = \frac{\text{Torq}/r_f}{W} , \quad (4-40)$$

where  $r_f$  is the mean seal radius.

Stiffness--The stiffness in the gas seal is computed numerically by varying  $h_0$  by,

$$S = \frac{dW}{dh_0} . \quad (4-41)$$

#### Example

The solution technique for the pressure has already been discussed in previous sections. Considering the total solution,  $h_0$  must be adjusted to satisfy equilibrium. Therefore,  $h_0$  becomes a dependent variable in the problem.



Specifically, the method of solution is:

- 1) Select a system of seal parameters and determine the applied load  $W^*$ .
- 2) Calculate the wear term  $w(r)$  using Equation (4-30).
- 3) Assume a value for minimum film thickness  $h_0$ .
- 4) By using the A.D.I. technique as described, the pressure distribution is solved for the given boundary conditions as described. A typical resultant pressure distribution is shown in Figure 4-10.
- 5) Calculate the total load support  $W$  using Equation (4-31).
- 6) Compare  $W$  to  $W^*$  and adjust  $h_0$  using a secant root finding method. Return to Step 4) and repeat until  $W = W^*$  within a desired accuracy.
- 7) Calculate coefficient of friction, leakage, percent load support and torque.

Appendix C contains a computer program listing for the solution to this problem.

Using the parameter values in Table 4-1, an example solution was obtained. The film thickness is similar to that shown in Figure 4-9. The values of pressure are shown in Figure 4-10. Pressure gradient in the outer edge of the seal is low and at the inner edge high. This suggests that accuracy of the leakage at the outer edge ( $Q_{out}$ ) is good while the accuracy at the inner edge ( $Q_{in}$ ) is low. It is much more difficult numerically to predict the steep gradient accurately. The load support required for equilibrium is 982.3 lbf (4.4 kN). The total load support determined by the fluid load and asperity load is found to be 982.26 lbf. Thus, equilibrium ( $W = W^*$ ) is attained. The minimum film thickness  $h_0$  is 30.2  $\mu$ in. (0.76  $\mu$ m). Since  $h_0 > c$  (0.5  $\mu$ m), no contact occurs. NITA, NITB and NITN are the numbers of iterations in the A.D.I. method required for the two initially guessed minimum film thicknesses and the equilibrium minimum film thickness respectively. The stiffness of the seal is

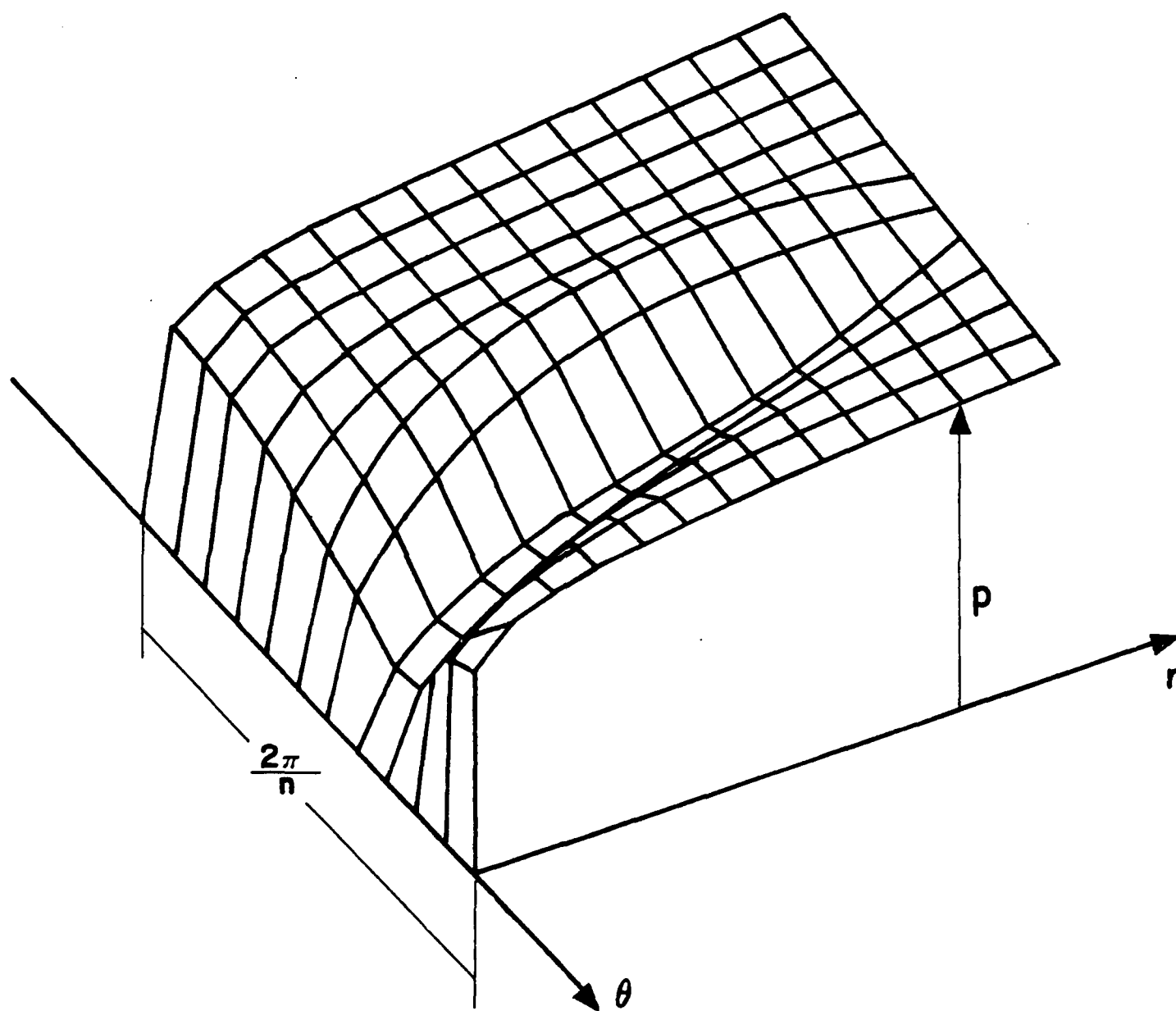


Figure 4-10. 3-D Plot of Pressure.

high ( $-1.15 \times 10^6$  lbf/in.) where the minus sign represents increasing load with decreasing gap. The leakage rate is 0.12 SCFM ( $3.4 \times 10^{-3}$  SCMM). Negative  $Q_{in}$  and  $Q_{out}$  represent leakage flow radially inward as expected for this example. The coefficient of friction is only 0.000045 which implies that the frictional force is small (only 0.043 lbf). One hundred percent fluid load support occurs. Total driving torque is only 0.088 in.-lb. Thus, both the leakage and torque are low. Using such results, the studies below were completed.

#### Parameter Studies

In this section parameter studies are made using the computational model developed. Four parameters, tilt, outside pressure, surface roughness, and speed have been considered in this initial study.

Tilt--Tilt is a design variable over which nearly complete control is possible. Increased tilt increases both radial taper and waviness. The effects of changing the tilt of a moving wave gas seal from negative 200 to positive 800  $\mu\text{m}/\text{m}$  are illustrated in Figure 4-11. The former value denotes a profile having a somewhat divergent radial taper over most of its circumferential period, while the latter value denotes a profile which has a predominantly convergent radial taper. These two extreme conditions are illustrated in Figure 4-12.

For negative and low values of tilt, the gas film stiffness, the coefficient of friction and the driving torque are relatively high, and at the same time the leakage rate is low indicating that mechanical or asperity-to-asperity contact has occurred. This is verified by the minimum film thickness  $h_0$  being less than the one-half maximum roughness height  $c$  as well as the percentage of the fluid film load support being less than 100 percent full film load support. However, as the tilt shifts in the

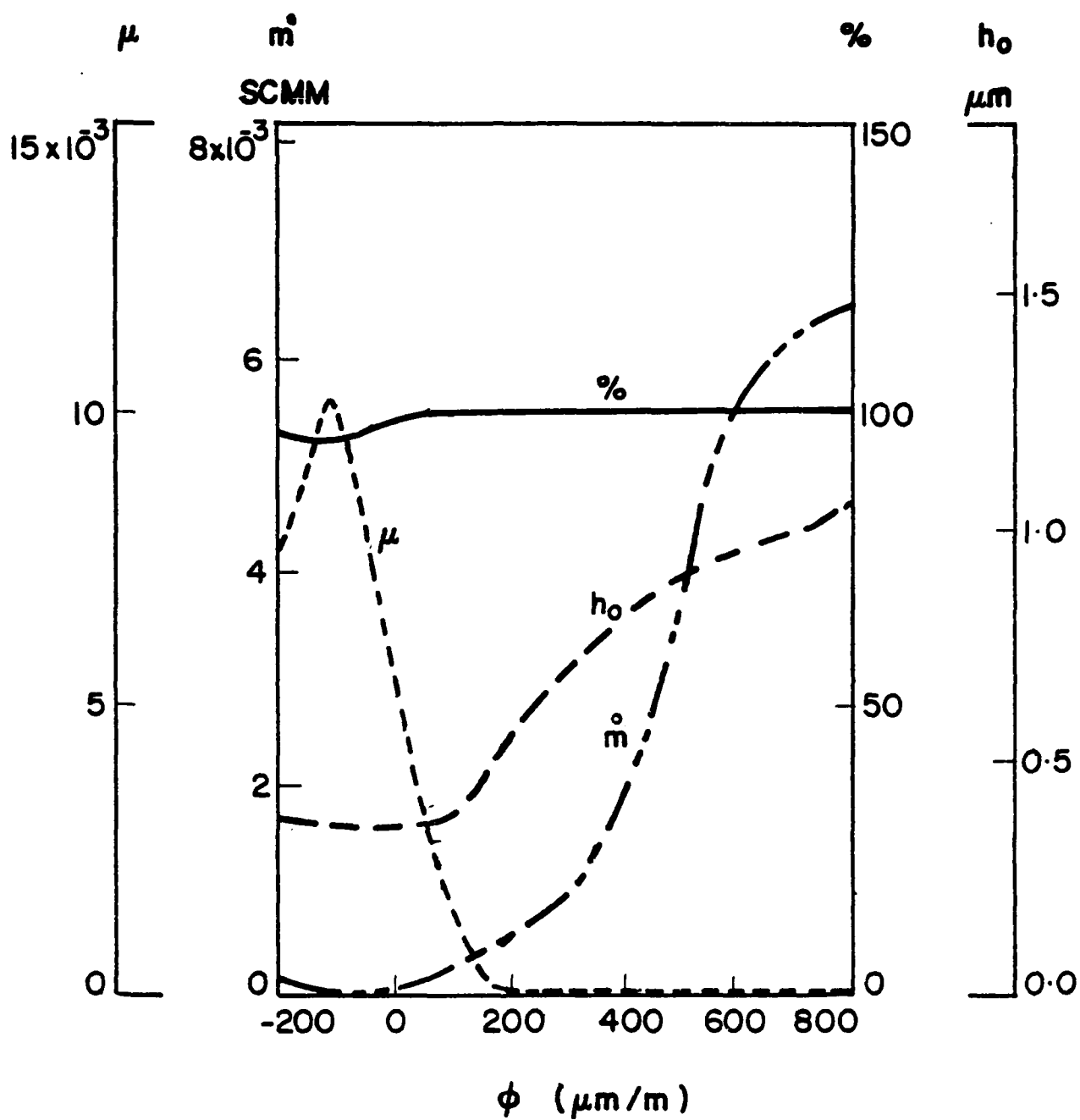
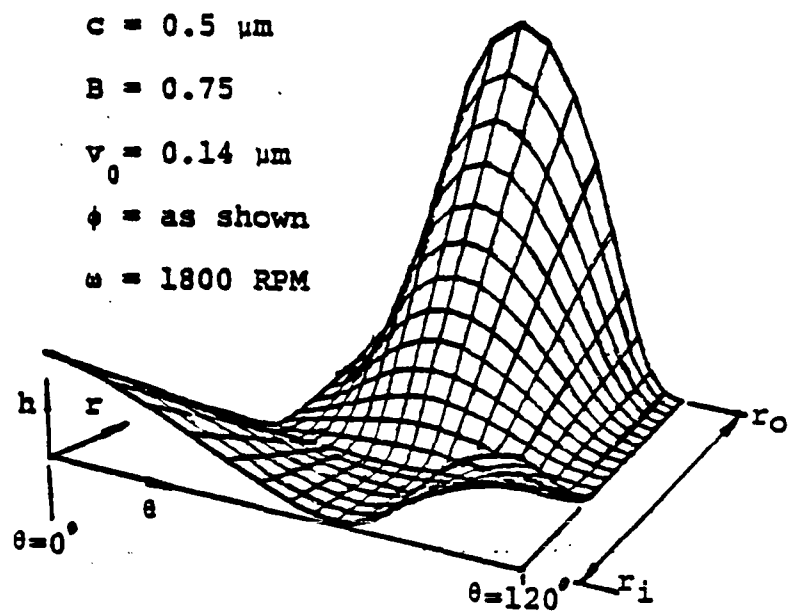
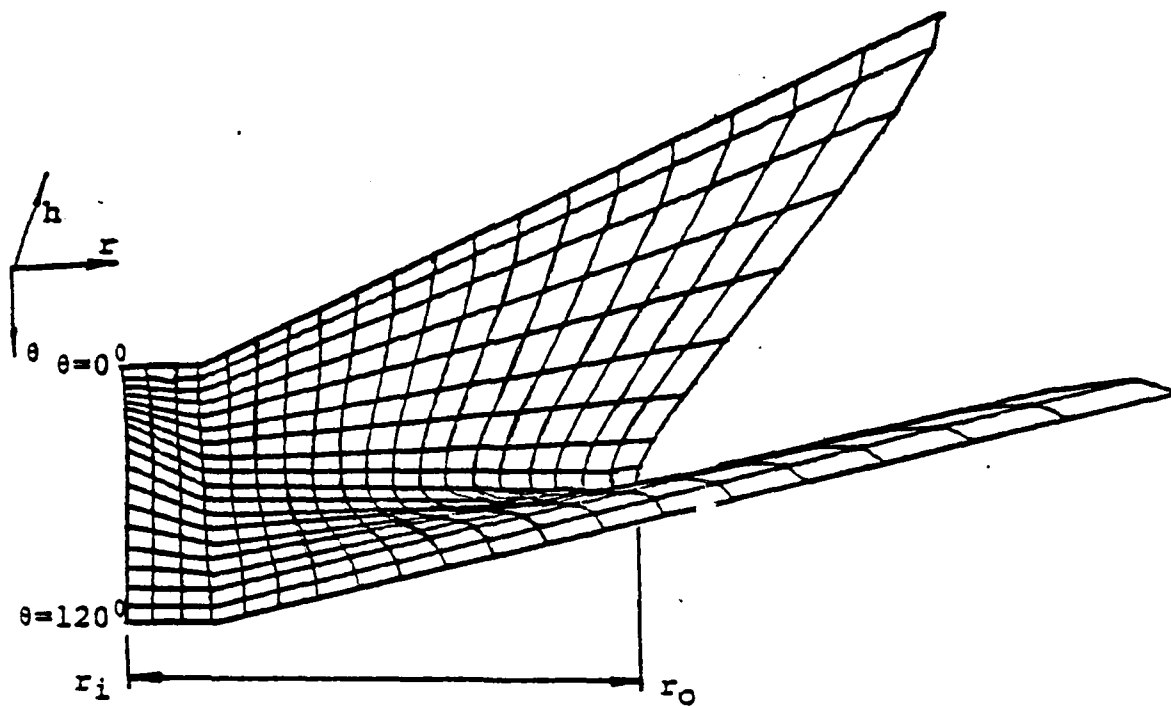


Figure 4-11. Effect of Tilt on Performance.



Tilt at  $-200 \text{ } \mu\text{m/m}$



Tilt at  $800 \text{ } \mu\text{m/m}$

Figure 4-12. Effect of Tilt on Film Thickness.

convergent direction, higher average fluid pressure results. The pressure distribution across the seal face has already been shown in Figure 4-10. As the tilt is increased the seal faces eventually separate, thus, the mean film thickness increases gradually and the leakage is higher. Thus, there is some minimum value of tilt increase to cause lift off.

Outside Pressure--The results of changing the outside pressure  $P_o$  while keeping the inside pressure  $P_i$  constant are shown in Figure 4-13. As the outside pressure increases from 0.0 to 1000 psig (0 to 7 MPa), the percentage of fluid load support increases rapidly to 100 percent. At low pressure there exists some mechanical asperity contact because the spring pressure is greater than the total lift force provided by hydrodynamic effects. The hydrostatic effect is zero at zero pressure.

Full film operation occurs when the pressure differential is high. As  $p_o$  approaches 200 psia, the minimum film thickness  $h_o$  becomes larger than the one-half maximum roughness height (c), and mechanical asperity-to-asperity contact no longer occurs. The hydrodynamic effect is not sufficient to lift off the seal against the spring load at zero sealed pressure.

In the gas seal analyzed the velocity of gas discharge is about 500 ft/sec under 1000 psig pressure differential (the worst case in Figure 4-13 is chosen) which is about 0.5 Mach number. The method of calculating discharge velocity is shown in Reference [23]. Therefore, sonic discharge does not occur. However, if the pressure differential keeps increasing, sonic flow will occur and the method of analysis will become quite complex according to [27].

Finally, the stiffness of the gaseous film is high when the outside pressure is less than 200 psig (1.4 MPa) because of mechanical contact.

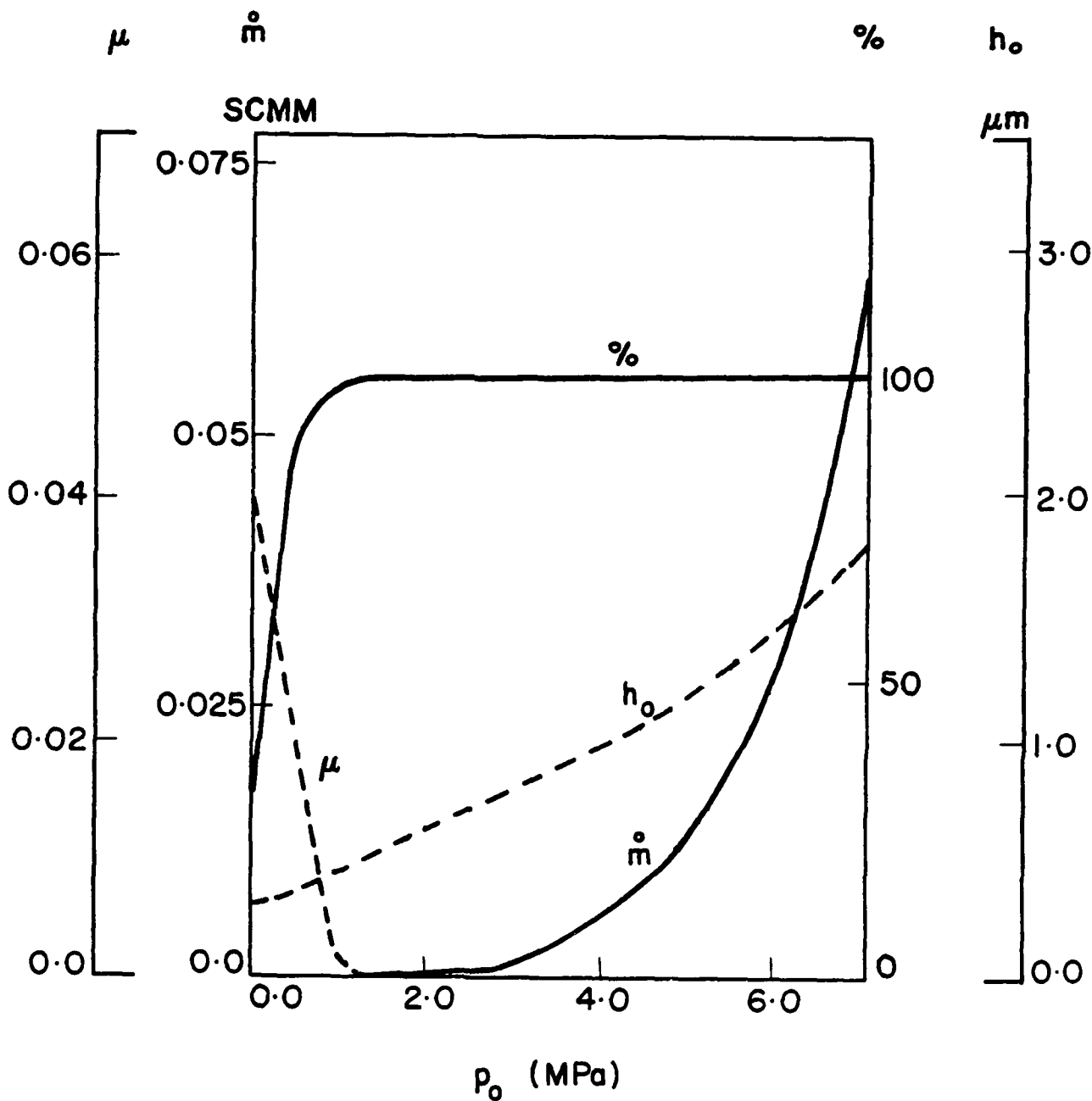


Figure 4-13. Effect of Outside Pressure.

Surface Roughness--The effect of surface roughness is shown in Figure 4-14. The performance parameters are quite constant when  $c$  is less than  $35 \mu\text{in.}$  ( $0.88 \mu\text{m}$ ). But as  $c$  increases, the friction begins to rapidly increase. The reason is that surface roughness  $c$  is now higher than the mean film thickness and thus asperity-to-asperity mechanical contact occurs.

Figure 4-15 shows the effect of surface roughness when there is no pressure difference between the two sides of the seal. Now, as the surface roughness decreases, the minimum film thickness decreases, the hydrodynamic effect becomes larger and larger until the seal lifts off (full film conditions). This is an important characteristic of the gas seal because if the seal cannot lift off at zero pressure difference, high frictional heat might occur at the contacting surfaces thus destroying the seal. The figure shows at  $n = 3$  that the maximum roughness allowable on the seal parts in order to provide the 95 percent lift off at zero pressure differential is in the order of  $c = 0.10 \mu\text{m}$  or  $4 \mu\text{in.}$  This corresponds to an RMS of about  $1.5 \mu\text{-in.}$  In practice, this surface roughness is difficult to achieve and maintain. Thus, it remains to be seen whether or not such a seal could survive operation at zero pressure differential. Clearly,  $n = 9$  would work better according to the figure. Also, a smaller spring load would help.

Angular Speed--The effect of angular velocity on performance parameters is illustrated in Figure 4-16. With the tilt at  $500 \mu\text{m/m}$ , there is no mechanical asperity contact even at zero speed. This and the fact that  $h_0$  does not significantly drop at low speed means that there is no significant hydrodynamic load support component. The entire load is supported by hydrostatic effects due to the radial taper  $\phi$ .

The coefficient of friction is approximately linearly proportional to the angular rotational speed  $\omega$  because the viscous drag force is proportional to angular speed.



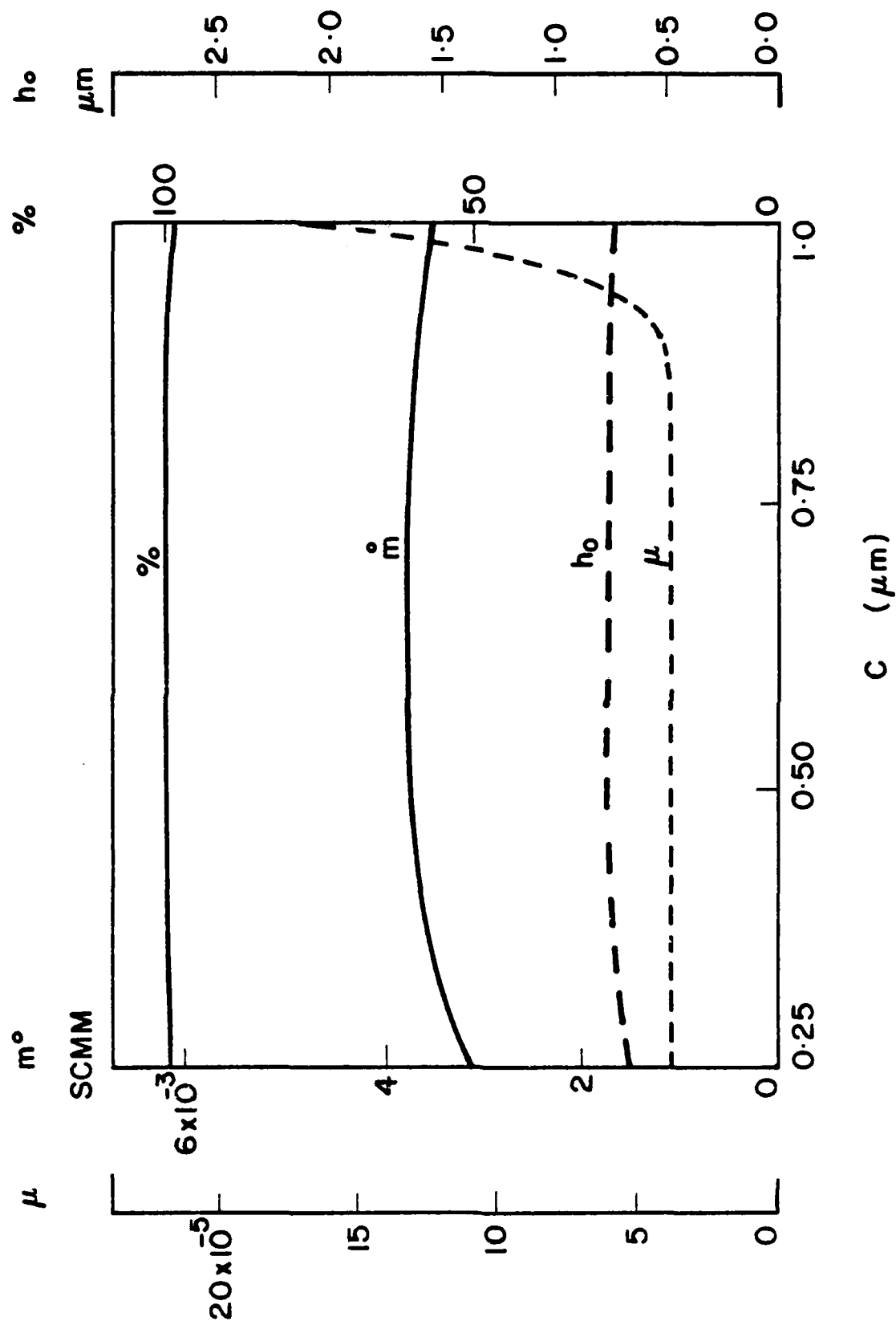


Figure 4-14. Effect of Surface Roughness.

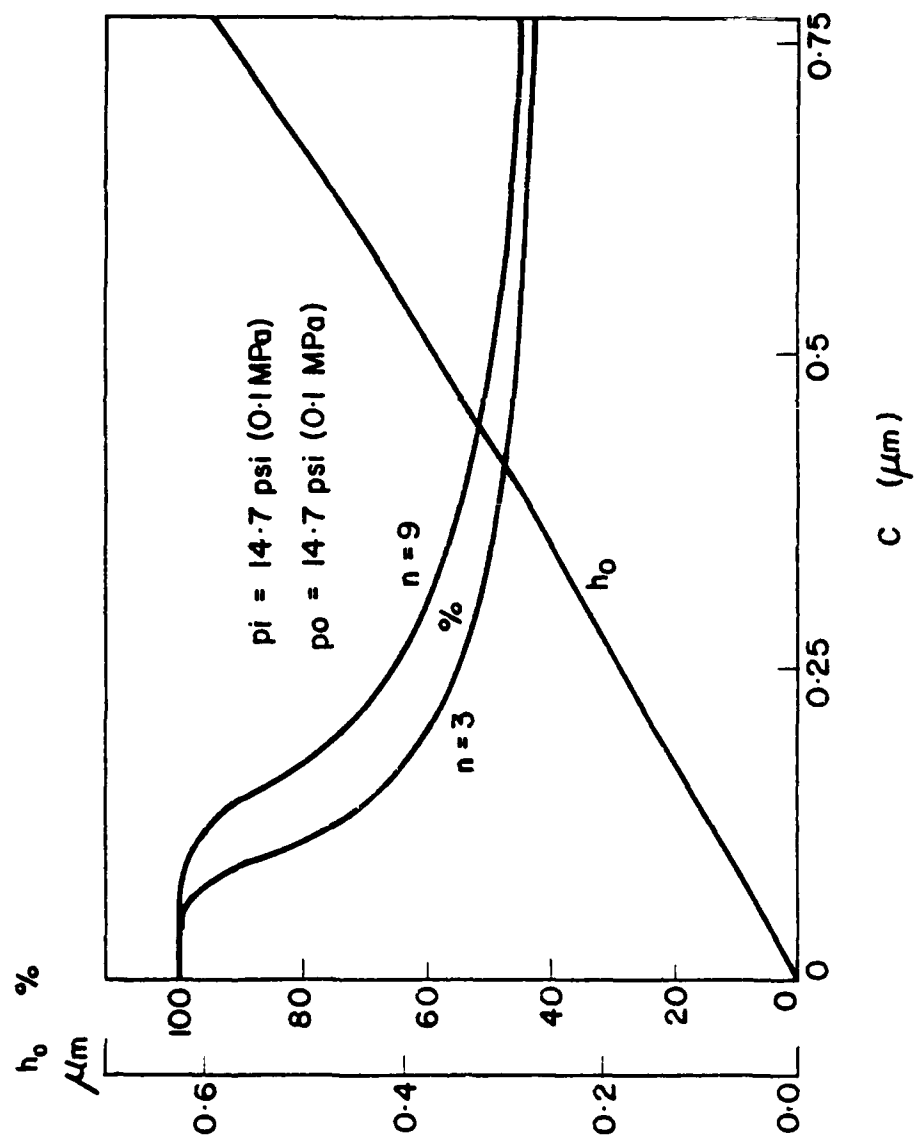


Figure 4-15. Hydrodynamic Effect.

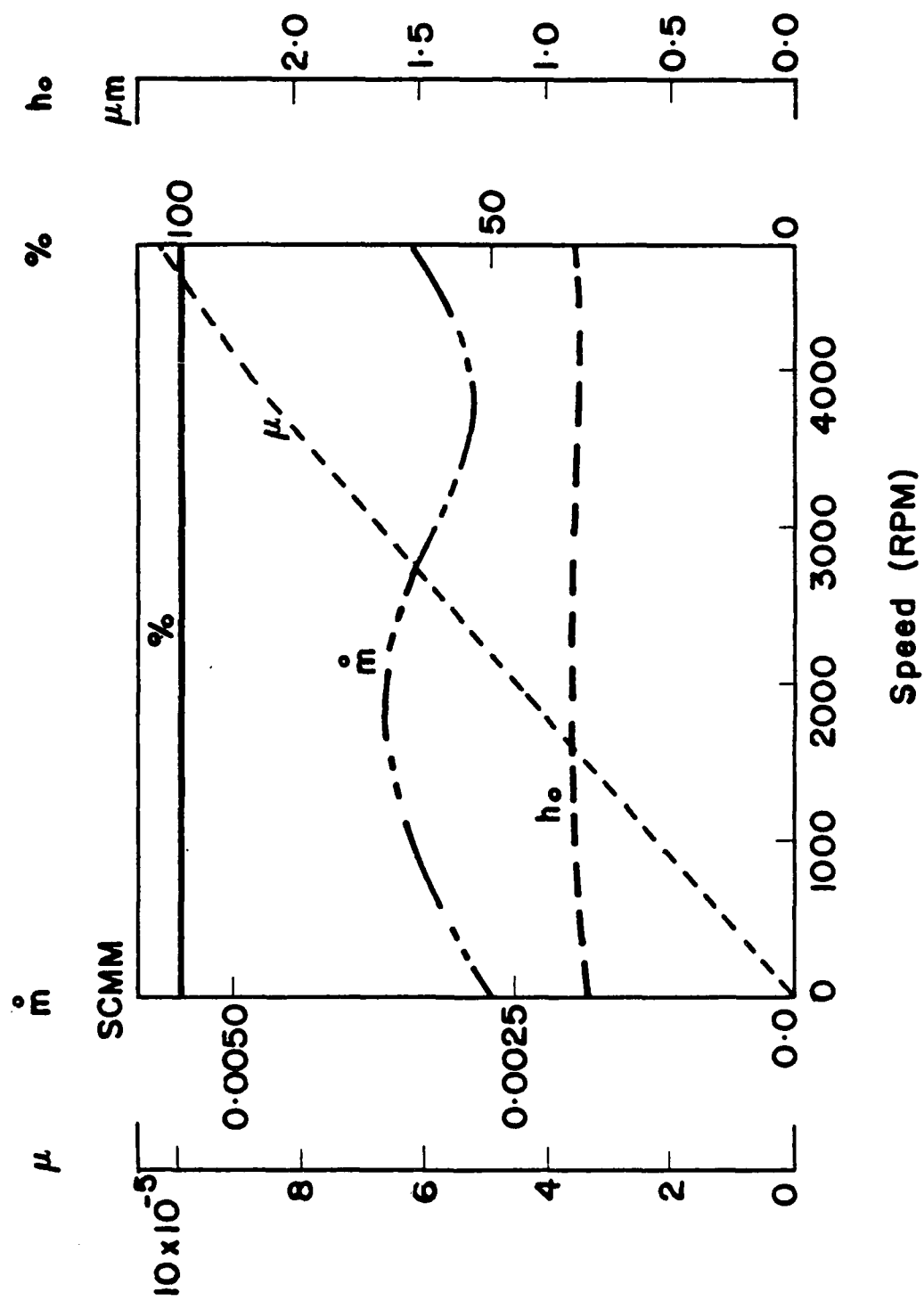


Figure 4-16. Effect of Angular Speed.

Potential Application--The preceding results show that a moving wave gas seal could theoretically operate at extremely low friction and wear with very low gas leakage. Load is supported primarily hydrostatically. The main question to be answered is whether or not sufficient hydrodynamic lift will develop under zero differential pressure conditions so as to avoid serious thermal distortion and/or wear of the seal. Theory shows that a flow surface roughness is essential to develop hydrodynamic load support. It is possible that a carbon surface would polish to the extent needed. Spring load could also be lowered. In any case, the answer to this question must be obtained experimentally.

## CHAPTER 5

### SEAL RING DEFLECTION

#### Background

It has long been recognized that seal ring faces may deflect out of their original lapped-in plane under the influence of non-axisymmetric loads, non-axisymmetric section properties, non-axisymmetric swelling of the carbon, or a combination of these factors. If such deflections are greater than those which can be flattened out by the axial load applied to the seal faces, unwanted gaps and leakage will result.

It is thought that one of the possible problems with present submarine seals is that the rings are too stiff to be able to close gaps created by shifting carbon segments. When gaps occur between seal faces, erosion continues to widen the gap until leakage becomes excessive. This potential problem or some variation of it affects the design of all seals including the design proposed and tested herein.

Thus, both for the purpose of analyzing problems with present seals and for designing new seals, some tool is needed to predict non-axisymmetric deflection for seal rings and also to predict potential gaps in seal ring pairs (the distribution of face loading). For simple rings with constant cross sectional properties, no splits, and simple harmonic deflections, the face loading distribution required for continuous contact is relatively easily obtained [41-42]. However, for more realistic conditions relating to the cross section properties and more complex distortions, finding the correct distribution of face loading becomes a very difficult problem.

The state of the art of predicting face loading in complex cases is illustrated by a report by Noell, Rippel, and Niemkiewicz of the Franklin Institute [43]. An evaluation of out-of-plane seal distortion caused by nonuniform joints was



made. It is shown how a nonuniform cross section near the joints causes out of flatness. In the report the contact between the seal faces is modeled by springs where tensile stresses across the faces are allowed. Since such stresses cannot occur in reality, the computed results do not predict how much and where the faces separate. Thus, the utility of the work to date is limited.

To serve as a useful tool in seal design a more exacting but general program is needed. Specifically, given two rings of arbitrary and circumferentially varying cross sections and load contacting on a plane and pressed together by axial loads, one must find the resulting face loading pressure distribution which arises as the rings are loaded up and distort. At regions where the faces do not contact, the resulting gap must be found. The same program would be capable of finding the resulting pressure and gap distribution given faces originally distorted out-of-plane as well.

There are two major features in the needed model. In the first place, one needs a very general tool to predict deflections in arbitrarily loaded variable cross section rings. Then, the two rings are brought together to contact and the pressure distribution is predicted. This second feature represents a specific type of contact problem. Suitable models for rings are not available in the literature and are developed herein. Several special ring models are developed and each one is tested against known solutions. The contact problem has been treated previously using finite element methods. A specific example which illustrates the technique is given here. Finally, it is shown how these models will be put together to find the needed solutions. All of the work is based on the finite elements techniques. Such a technique is essential for the contact problem.

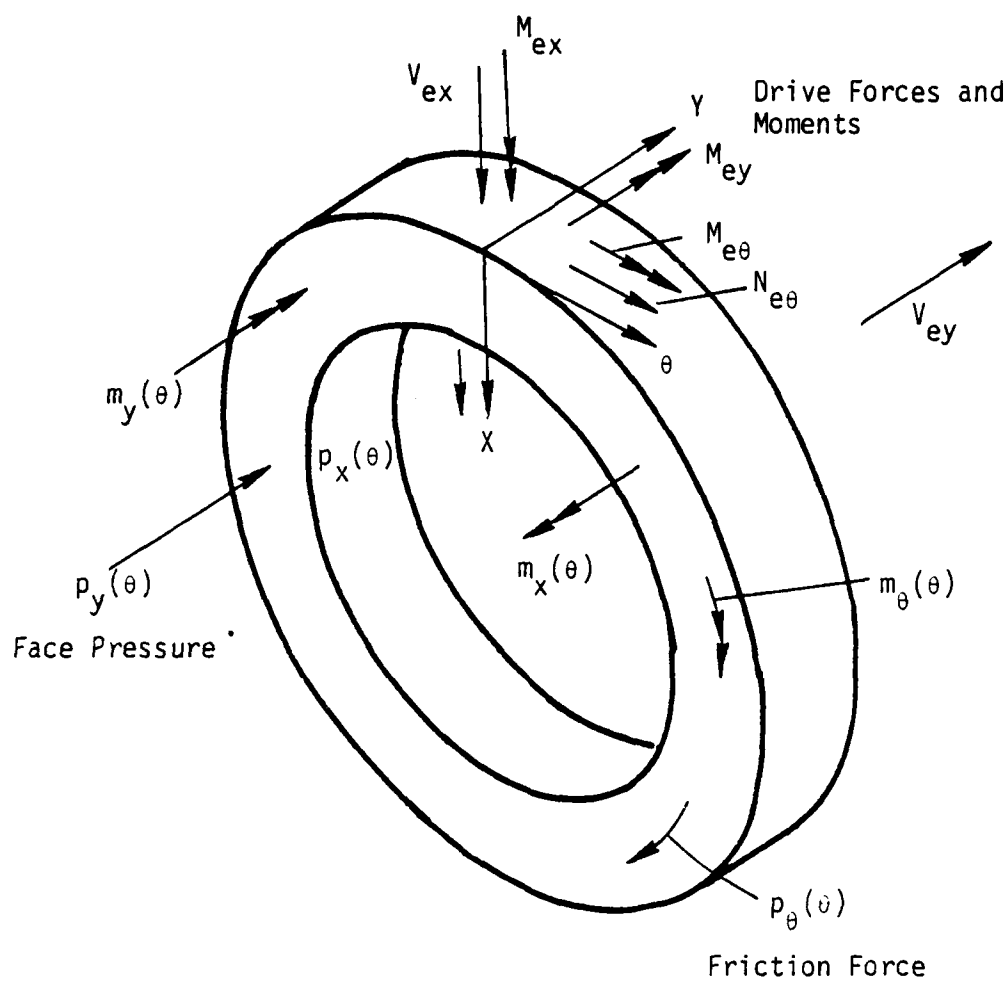


Figure 5-1. Mechanical Loading on a Seal Ring.

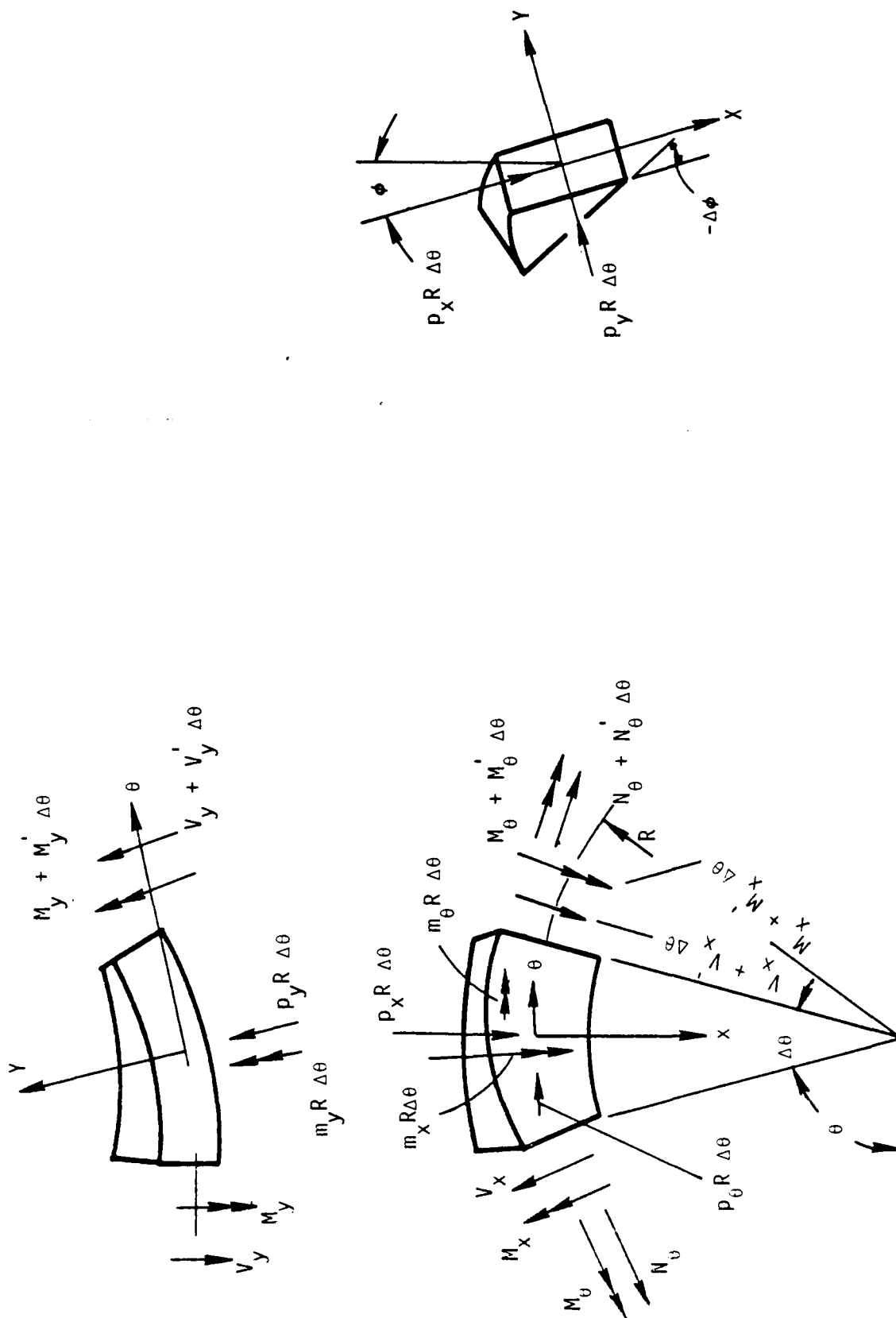


Figure 5-2. Segment of a Ring.



deflections. Based on the previous work [41-42] and neglecting all distributed loads (these will be added later at the nodes), the equations of equilibrium are:

$$V'_x + N_\theta = 0^* \quad (5-1)$$

$$V'_y = 0 \quad (5-2)$$

$$N'_\theta - V_x = 0 \quad (5-3)$$

$$M'_x + M_\theta - V_y R = 0 \quad (5-4)$$

$$M'_y + R V_x = 0 \quad (5-5)$$

$$M'_\theta - M_x = 0 \quad (5-6)$$

The above equations assume the distance between the shear center and the centroid is negligibly small. Combining the above, the equilibrium equations become

$$N''_\theta = N_\theta = 0 \quad (5-7)$$

$$M''_x + M_\theta = 0 \quad (5-8)$$

$$M''_y + M_y = 0 \quad (5-9)$$

$$M'_\theta - M_x = 0 \quad (5-10)$$

Stress resultant-displacement equations are given by

$$N_\theta + \frac{M_y}{R} = \frac{Ea}{R} [w' - u] \quad (5-11)$$

$$M_x = \frac{E}{R^2} [(\phi R - v'') J_x - (u'' + u) J_{xy}] \quad (5-12)$$

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\*Primes denote derivatives with respect to  $\theta$ .

$$M_y = \frac{E}{R^2} [(u + u'') J_y - (\phi R - v'') J_{xy}] \quad (5-13)$$

$$M_\theta = \frac{G}{R^2} J_\theta [\phi' R + v'] \quad (5-14)$$

where

$$J_x = \int_a \frac{y^2}{1 - x/R} da \quad (5-15)$$

$$J_y = \int_a \frac{x^2}{1 - x/R} da \quad (5-16)$$

$$J_{xy} = \int_a \frac{xy}{1 - x/R} da \quad (5-17)$$

$$J_\theta = \text{torsional stiffness constant such as in Oden [45]} \quad (5-18)$$

$$a = \int_a da \quad (5-19)$$

Substituting Equations (5-11) through (5-14) into Equations (5-7) through (5-10) results in the following differential equations describing deflections:

$$\begin{aligned} w' + w''' - (u + u'') \left(1 + \frac{J_y}{aR^2}\right) - (u'' + u''') \frac{J_y}{aR^2} \\ + (\phi R + \phi'' R - v'' - v''') \frac{J_{xy}}{aR^2} = 0 \end{aligned} \quad (5-20)$$

$$\phi'' R \left(1 + \frac{GJ_\theta}{EJ_x}\right) - v'''' + v'' \frac{GJ_\theta}{EJ_x} - (u'''' + u'') \frac{J_{xy}}{J_x} = 0 \quad (5-21)$$

$$(u' + 2u''' + u''''') J_y - (\phi'R + \phi'''R - v''' - v''''') J_{xy} = 0 \quad (5-22)$$

$$(\phi''R + v'') \frac{GJ_\theta}{EJ_x} - (\phi R - v') + (u'' + u) \frac{J_{xy}}{J_x} = 0 \quad (5-23)$$

It can be seen if  $J_{xy} = 0$ , then the out-of-plane displacements  $v$ ,  $\phi$  are decoupled from the in-plane displacements  $u$ ,  $w$ . The above equations consist of four differential equations in the unknowns  $u$ ,  $v$ ,  $w$ ,  $\phi$ .

#### Finite Element

Given the governing deflection equations, one can now derive a finite element stiffness matrix for a general ring element. The stiffness matrix is defined by

$$[F] = [K] [\delta] . \quad (5-24)$$

That is, given an arbitrary set of displacements at the end of the element  $\delta$ , what are the forces and moments necessary.  $[K]$ , the stiffness matrix, defines this characteristic and is all that is required to define a finite element.

There are various coordinate systems in which the stiffness matrix can be defined. In this work, a polar coordinate system was chosen. This makes the stiffness matrix invariant around the ring assuming constant section properties. Thus the stiffness matrix only needs to be computed one time for a given element size. It can then be used at any location around the ring without using a coordinate transformation as is required for other stiffness matrices such as those presented by Morris [49].

Three different stiffness matrices will now be derived, one for in-plane bending, one for out-of-plane bending, and one for the coupled problem. The two simpler cases are used to provide a check on the limiting cases of the coupled problem of interest. The solution of the two simple cases also helps in understanding the solution to the coupled problem.

In-Plane Stiffness--Figure 5-3 shows an element defined for in-plane stiffness. For the in-plane case, Equations (5-20) and (5-22) for  $J_{xy} = 0$  reduce to the following

$$w' + w''' - (u + u'') \left( 1 + \frac{J_y}{aR^2} \right) - (u'' + u''') \frac{J_y}{aR^2} = 0, \quad (5-25)$$

$$u' + 2u''' + u'''' = 0. \quad (5-26)$$

The objective is to solve the above two equations subject to the displacement boundary conditions shown in Figure 5-3.

The general solution to Equation (5-26) is

$$u = C_1 + C_2 \sin \theta + C_3 \cos \theta + C_4 \theta \sin \theta + C_5 \theta \cos \theta. \quad (5-27)$$

Substitution of (5-27) into (5-25) and solving for  $w$  gives

$$w = C_6 + C_7 \sin \theta + C_8 \cos \theta + C_1 \left( \frac{J_y}{aR^2} + 1 \right) \theta - C_4 \theta \cos \theta + C_5 \theta \sin \theta. \quad (5-28)$$

There are six displacement boundary conditions shown in Figure 5-3 and eight arbitrary constants above. It follows that there are too many constants and that substitution back into the original differential equation will eliminate two of the constants.

Using Equations (5-11), (5-12), (5-27), and (5-28), one may show that

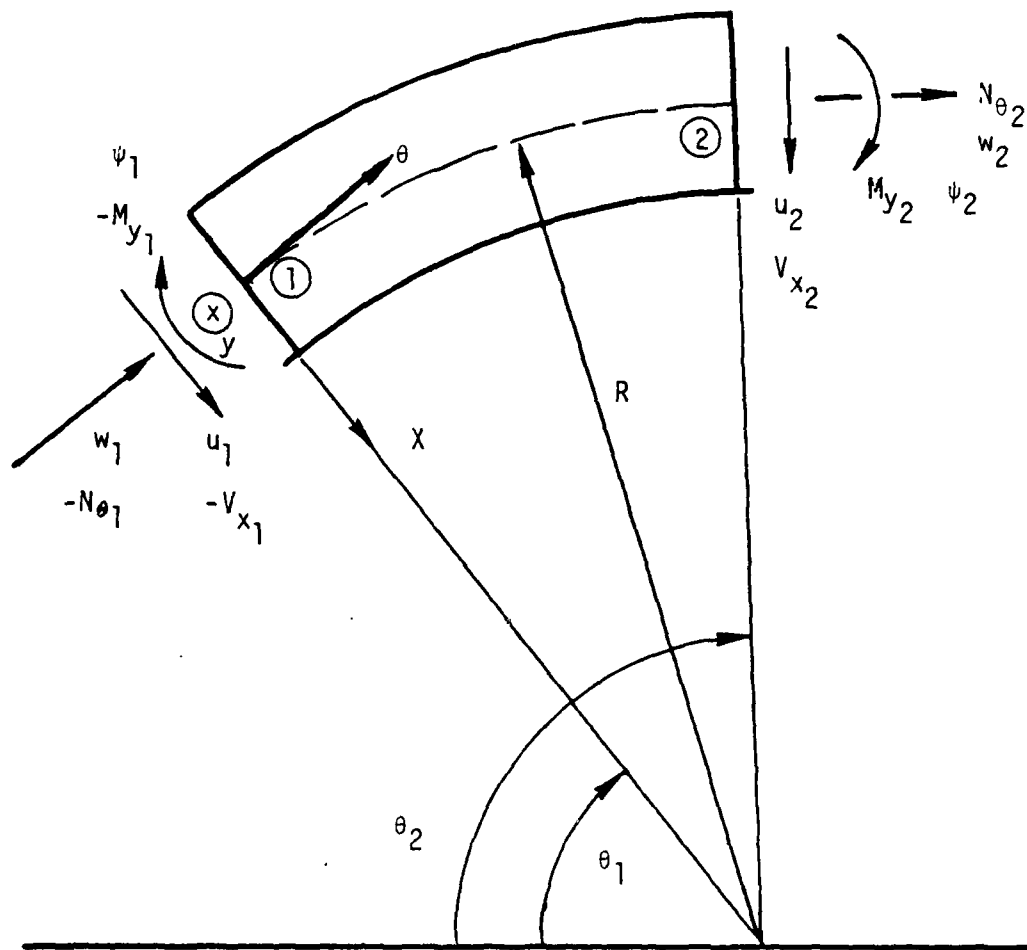


Figure 5-3. Finite Element for In-plane Loading.

$$N_{\theta} = \frac{Ea}{R} [C_7 \cos \theta - C_8 \sin \theta - C_4 \cos \theta + C_5 \sin \theta - C_2 \sin \theta - C_3 \cos \theta] - \frac{EJ_y}{R^3} (2C_4 \cos \theta - 2C_5 \sin \theta) \quad (5-29)$$

Similarly,

$$M_y = \frac{EJ_y}{R^2} (C_1 + 2C_4 \cos \theta - 2C_5 \sin \theta) . \quad (5-30)$$

Going back to the original equilibrium equations, Equations (5-3) and (5-5) show that

$$M_y' = -RN_{\theta}' . \quad (5-31)$$

Substituting (5-29) and (5-30) into (5-31) shows that

$$C_8 = C_5 - C_2 \quad (5-32)$$

$$C_7 = C_4 + C_3 \quad (5-33)$$

so that two of the constants are eliminated.

The boundary conditions are shown in Figure 5-3. Rotation  $\psi$  represents the total in-plane rotation which is compatible with  $M_y$ .

$$\psi = \frac{1}{R} \frac{du}{d\theta} + \frac{w}{R} . \quad (5-34)$$

The negative forces and moments shown at end ① of the beam elements are in the same direction as the displacements as required for the stiffness matrix.

Using Equations (5-27), (5-28), (5-32) and (5-33) and summarizing

$$u = C_1 + C_2 \sin \theta + C_3 \cos \theta + C_4 \theta \sin \theta + C_5 \theta \cos \theta, \quad (5-35)$$

$$\psi = \frac{1}{R} \left[ C_1 \left( \frac{J_y}{aR^2} + 1 \right) \theta + 2C_4 \sin \theta + 2C_5 \cos \theta + C_6 \right], \quad (5-36)$$

$$w = C_1 \left( \frac{J_y}{aR^2} + 1 \right) \theta - C_2 \cos \theta + C_3 \sin \theta + C_4 (\sin \theta - \theta \cos \theta) \\ + C_5 (\cos \theta + \theta \sin \theta) + C_6. \quad (5-37)$$

Thus, to solve for the constants  $C_i$  boundary conditions as shown are applied to Equations (5-35), (5-36) and (5-37) as follows.

$$\begin{bmatrix} 1 & \sin \theta_1 & \cos \theta_1 & \theta_1 \sin \theta_1 & \theta_1 \cos \theta_1 & 0 \\ \left( \frac{J_y}{aR^2} + 1 \right) \frac{\theta_1}{R} & 0 & 0 & \frac{2 \sin \theta_1}{R} & \frac{2 \cos \theta_1}{R} & \frac{1}{R} \\ \left( \frac{J_y}{aR^2} + 1 \right) \theta_1 & -\cos \theta_1 & \sin \theta_1 & \sin \theta_1 - \theta_1 \cos \theta_1 & \cos \theta_1 + \theta_1 \sin \theta_1 & 1 \\ 1 & \sin \theta_2 & \cos \theta_2 & \theta_2 \sin \theta_2 & \theta_2 \cos \theta_2 & 0 \\ \left( \frac{J_y}{aR^2} + 1 \right) \frac{\theta_2}{R} & 0 & 0 & \frac{2 \sin \theta_2}{R} & \frac{2 \cos \theta_2}{R} & \frac{1}{R} \\ \left( \frac{J_y}{aR^2} + 1 \right) \theta_2 & -\cos \theta_2 & \sin \theta_2 & \sin \theta_2 - \theta_2 \cos \theta_2 & \cos \theta_2 + \theta_2 \sin \theta_2 & 1 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} = \begin{bmatrix} u_1 \\ \psi_1 \\ w_1 \\ u_2 \\ \psi_2 \\ w_2 \end{bmatrix} \quad (5-38)$$

or

$$[A_{in}] [C] = [\delta_{in}]. \quad (5-39)$$

Now, using Equations (5-5), (5-29), and (5-30) one may summarize the forces as follows.

$$V_x = \frac{EJ_y}{R^3} (2C_4 \sin \theta + 2C_5 \cos \theta) , \quad (5-40)$$

$$M_y = \frac{EJ_y}{R^2} (C_1 + 2C_4 \cos \theta - 2C_5 \sin \theta) , \quad (5-41)$$

$$N_\theta = \frac{EJ_y}{R^3} (-2C_4 \cos \theta + 2C_5 \sin \theta) . \quad (5-42)$$

The generalized force vector is given by

$$\frac{EJ_y}{R^2} \begin{bmatrix} 0 & 0 & 0 & -\frac{2}{R} \sin \theta_1 & -\frac{2}{R} \cos \theta_1 & 0 \\ -1 & 0 & 0 & -2 \cos \theta_1 & 2 \sin \theta_1 & 0 \\ 0 & 0 & 0 & \frac{2}{R} \cos \theta_1 & -\frac{2}{R} \sin \theta_1 & 0 \\ 0 & 0 & 0 & \frac{2}{R} \sin \theta_2 & \frac{2}{R} \cos \theta_2 & 0 \\ 1 & 0 & 0 & 2 \cos \theta_2 & -2 \sin \theta_2 & 0 \\ 0 & 0 & 0 & -\frac{2}{R} \cos \theta_2 & \frac{2}{R} \sin \theta_2 & 0 \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{bmatrix} = \begin{bmatrix} -V_{x1} \\ -M_{y1} \\ -N_{\theta1} \\ V_{x2} \\ M_{y2} \\ N_{\theta2} \end{bmatrix} \quad (5-43)$$

or

$$\frac{EJ_y}{R^2} [D_{in}] [C] = [F_{in}] . \quad (5-44)$$

From (5-39) and (5-44)



$$[F_{in}] = \frac{EJ_y}{R^2} [D_{in}] [A_{in}]^{-1} [\delta_{in}] \quad (5-45)$$

so

$$[K_{in}] = \frac{EJ_y}{R^2} [D_{in}] [A_{in}]^{-1} \quad (5-46)$$

To provide an example for future reference, the following parameter values are selected

$$R = 3.0 \text{ in}$$

$$\frac{J_y}{aR^2} = 0.01 \quad (5-47)$$

$$\theta_2 = 0$$

$$\theta_2 = \pi/6$$

For this example the stiffness matrix  $[K]$  is:

$$[K_{in}] = \frac{EJ_y}{R^2} \begin{bmatrix} 30.72 & 21.84 & 9.21 & 22.00 & 20.54 & 23.34 \\ 21.89 & 22.80 & -3.24 & -20.54 & 11.26 & -8.11 \\ 9.21 & -3.24 & 62.62 & 23.34 & -8.11 & -58.84 \\ -22.00 & -20.54 & 23.34 & 30.72 & -21.84 & -9.21 \\ 20.54 & 11.26 & -8.11 & -21.84 & 22.80 & -3.24 \\ -23.34 & -8.11 & -58.84 & -9.21 & -3.24 & 62.62 \end{bmatrix} \quad (5-48)$$

The above equations have been checked against those given by Morris [49]. The stiffness matrix is invariant with orientation  $\theta_1$  given a constant  $\theta_2 - \theta_1$  as it should be.

Out-of-Plane Stiffness--Figure 5-4 shows an element defined for out-of-plane stiffness. The moments and forces are shown which are compatible with the displacements. The stiffness for this element is determined in a manner similar to that for the in-plane case. Letting  $J_{xy} \rightarrow 0$ , the out-of-plane equations are taken from Equation (5-21) and (5-23).

$$\phi''R(1 + A) - v''''A + v'' = 0 , \quad (5-49)$$

$$(\phi''R + v'') - A(\phi R - v'') = 0 , \quad (5-50)$$

where

$$A = \frac{EJ_x}{GJ_\theta} . \quad (5-51)$$

Elimination of  $\phi$  between (5-49) and (5-50) gives

$$v'''''' + 2v'''' + v'' = 0 \quad (5-52)$$

to which the general solution is

$$v = B_1 + B_2 \theta + B_3 \sin \theta + B_4 \cos \theta + B_5 \theta \sin \theta + B_6 \theta \cos \theta . \quad (5-53)$$

Using (5-49) to solve for  $\phi$  gives

$$\begin{aligned} \phi R = & -B_3 \sin \theta - B_4 \cos \theta - B_5 \theta \sin \theta - B_6 \theta \cos \theta \\ & + 2B_5 \frac{A}{1+A} \cos \theta - 2B_6 \frac{A}{1+A} \sin \theta + B_7 \theta + B_8 . \end{aligned} \quad (5-54)$$

Substitution into (5-50) shows that

$$B_7 = 0 , \quad B_8 = 0 . \quad (5-55)$$

Thus, the solutions for  $\phi$  and  $v$  are now given.

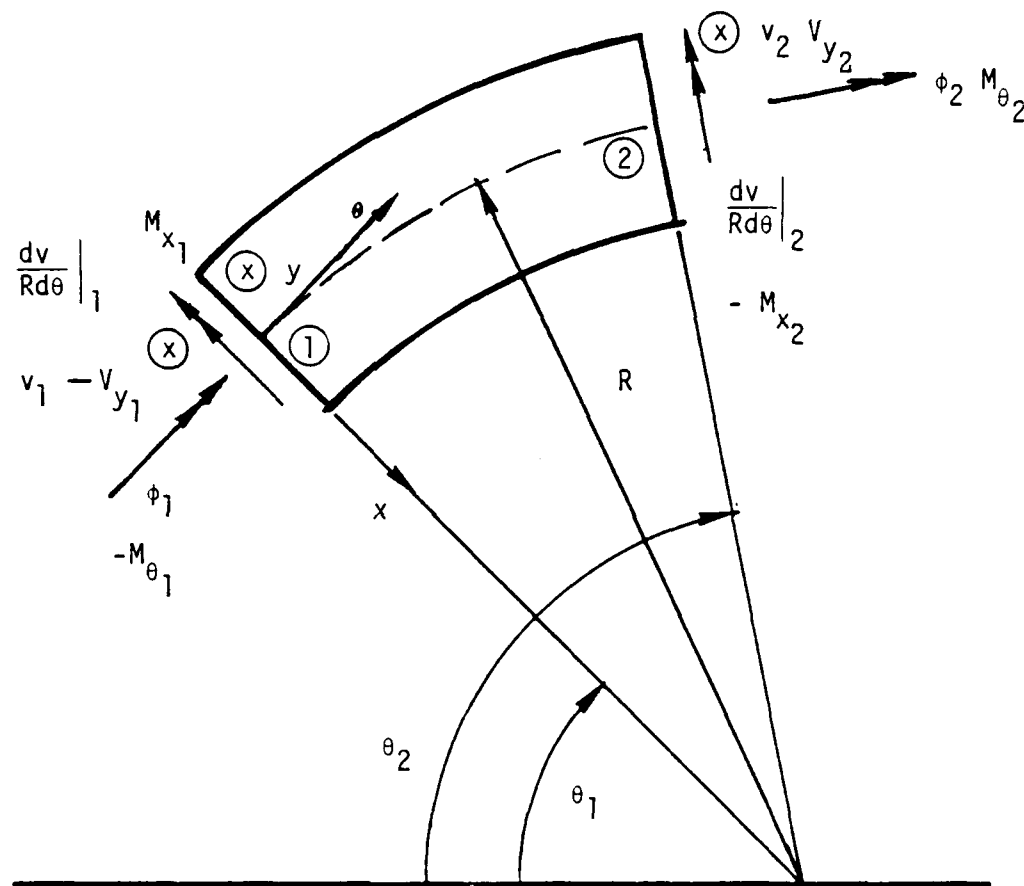


Figure 5-4. Finite Element for Out-of-Plane Loading.

Applying the boundary conditions of Figure 5-4 to Equations (5-53) and (5-54) gives the following matrix equations

$$\begin{bmatrix}
 1 & \theta_1 & \sin \theta_1 & \cos \theta_1 & \theta_1 \sin \theta_1 & \theta_1 \cos \theta_1 \\
 0 & \frac{1}{R} & \frac{1}{R} \cos \theta_1 & -\frac{1}{R} \sin \theta_1 & \frac{1}{R} \sin \theta_1 + \frac{1}{R} \theta_1 \cos \theta_1 & \frac{1}{R} \cos \theta_1 - \frac{1}{R} \theta_1 \sin \theta_1 \\
 0 & 0 & -\frac{1}{R} \sin \theta_1 & -\frac{1}{R} \cos \theta_1 & -\frac{\theta_1}{R} \sin \theta_1 + \frac{2}{R} \frac{A}{1+A} \cos \theta & -\frac{\theta_1}{R} \cos \theta_1 - \frac{2}{R} \frac{A}{1+A} \sin \theta_1 \\
 1 & \theta_2 & \sin \theta_2 & \cos \theta_2 & \theta_2 \sin \theta_2 & \theta_2 \cos \theta_2 \\
 0 & \frac{1}{R} & \frac{1}{R} \cos \theta_2 & -\frac{1}{R} \sin \theta_2 & \frac{1}{R} \sin \theta_2 + \frac{1}{R} \theta_2 \cos \theta_2 & \frac{1}{R} \cos \theta_2 - \frac{1}{R} \theta_2 \sin \theta_2 \\
 0 & 0 & -\frac{1}{R} \sin \theta_2 & -\frac{1}{R} \cos \theta_2 & -\frac{\theta_2}{R} \sin \theta_2 + \frac{2}{R} \frac{A}{1+A} \cos \theta_2 & -\frac{\theta_2}{R} \cos \theta_2 - \frac{2}{R} \frac{A}{1+A} \sin \theta_2
 \end{bmatrix}
 \begin{bmatrix}
 B_1 \\
 B_2 \\
 B_3 \\
 B_4 \\
 B_5 \\
 B_6
 \end{bmatrix}
 =
 \begin{bmatrix}
 v_1 \\
 \frac{1}{R} \frac{dv}{d\theta} \Big|_1 \\
 \phi_1 \\
 v_2 \\
 \frac{1}{R} \frac{dv}{d\theta} \Big|_2 \\
 \phi_2
 \end{bmatrix}
 \quad (5-56)$$

or

$$[A_{out}] [B] = [\delta_{out}] \quad (5-57)$$

Using Equations (5-4), (5-13), and (5-14) and substituting for  $v$  and  $\phi$ , one can show that

$$V_y = \frac{EJ_x}{R^3} \frac{B_2}{A}, \quad (5-58)$$

$$M_x = \frac{EJ_x}{R^2} \left( \frac{2}{1+A} \right) (-B_5 \cos \theta + B_6 \sin \theta) \quad (5-59)$$

$$M_\theta = \frac{EJ_x}{R^2} \left( \frac{B_2}{A} - \frac{2B_5}{1+A} \sin \theta - \frac{2B_6}{1+A} \cos \theta \right). \quad (5-60)$$

Thus the generalized force vector is given by

$$\frac{EJ_x}{R^2} \begin{bmatrix} 0 & -\frac{1}{RA} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{2}{1+A} \cos \theta_1 & \frac{2}{1+A} \sin \theta_1 \\ 0 & \frac{1}{A} & 0 & 0 & \frac{2}{1+A} \sin \theta_1 & \frac{2}{1+A} \cos \theta_1 \\ 0 & \frac{1}{RA} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2}{1+A} \cos \theta_2 & -\frac{2}{1+A} \sin \theta_2 \\ 0 & \frac{1}{A} & 0 & 0 & -\frac{2}{1+A} \sin \theta_2 & -\frac{2}{1+A} \cos \theta_2 \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{bmatrix} = \begin{bmatrix} -V_{y1} \\ M_{x1} \\ -M_{\theta 1} \\ V_{y2} \\ -M_{x2} \\ M_{\theta 2} \end{bmatrix} \quad (5-61)$$

or

$$\frac{EJ_x}{R^2} [D_{out}] [B] = [F_{out}] \quad (5-62)$$

From Equations (5-57) and (5-62)

$$[F_{out}] = \frac{EJ_x}{R^2} [D_{out}] [A_{out}]^{-1} [\delta_{out}] \quad (5-63)$$

so

$$[K_{out}] = \frac{EJ_x}{R^2} [D_{out}] [A_{out}]^{-1} \quad (5-64)$$

Using as an example

$$R = 3.0 \text{ in.}$$

$$A = 5.0$$

$$\theta_1 = 0 \quad (5-65)$$

$$\theta_2 = \pi/6 ,$$

The stiffness matrix for the out-of-plane case is:

$$[K_{out}] = \frac{EJ_x}{R^2} \begin{bmatrix} 27.12 & -21.20 & 2.24 & -27.12 & 21.20 & -2.24 \\ 21.20 & 21.55 & 2.77 & -21.20 & 11.75 & -0.15 \\ 2.24 & 2.77 & 1.63 & -2.24 & 0.15 & -0.92 \\ -27.12 & -21.20 & -2.24 & 27.12 & -21.20 & 2.24 \\ 21.20 & 11.75 & 0.15 & -21.20 & 21.55 & -2.77 \\ -2.24 & -0.15 & -0.92 & 2.24 & -2.77 & 1.63 \end{bmatrix} \quad (5-66)$$

Coupled Stiffness--Figure 5-5 shows an element defined for both in- and out-of-plane stiffness. The forces and displacements defined for the two previous cases are shown together in Figure 5-5.

To find the stiffness for this element, the set of coupled Equations (5-20) through (5-23) must be solved subject to the general displacement boundary conditions shown on Figure 5-5. Eliminating  $u$  and  $\phi$  between Equations (5-21), (5-22), and (5-23) an equation in  $v$  only is found.

$$v'''''' + 2v'''' + v'' = 0 . \quad (5-67)$$

As before, the general solution is

$$v = G_1 + G_2 \theta + G_3 \sin \theta + G_4 \cos \theta + G_5 \theta \sin \theta + G_6 \theta \cos \theta . \quad (5-68)$$

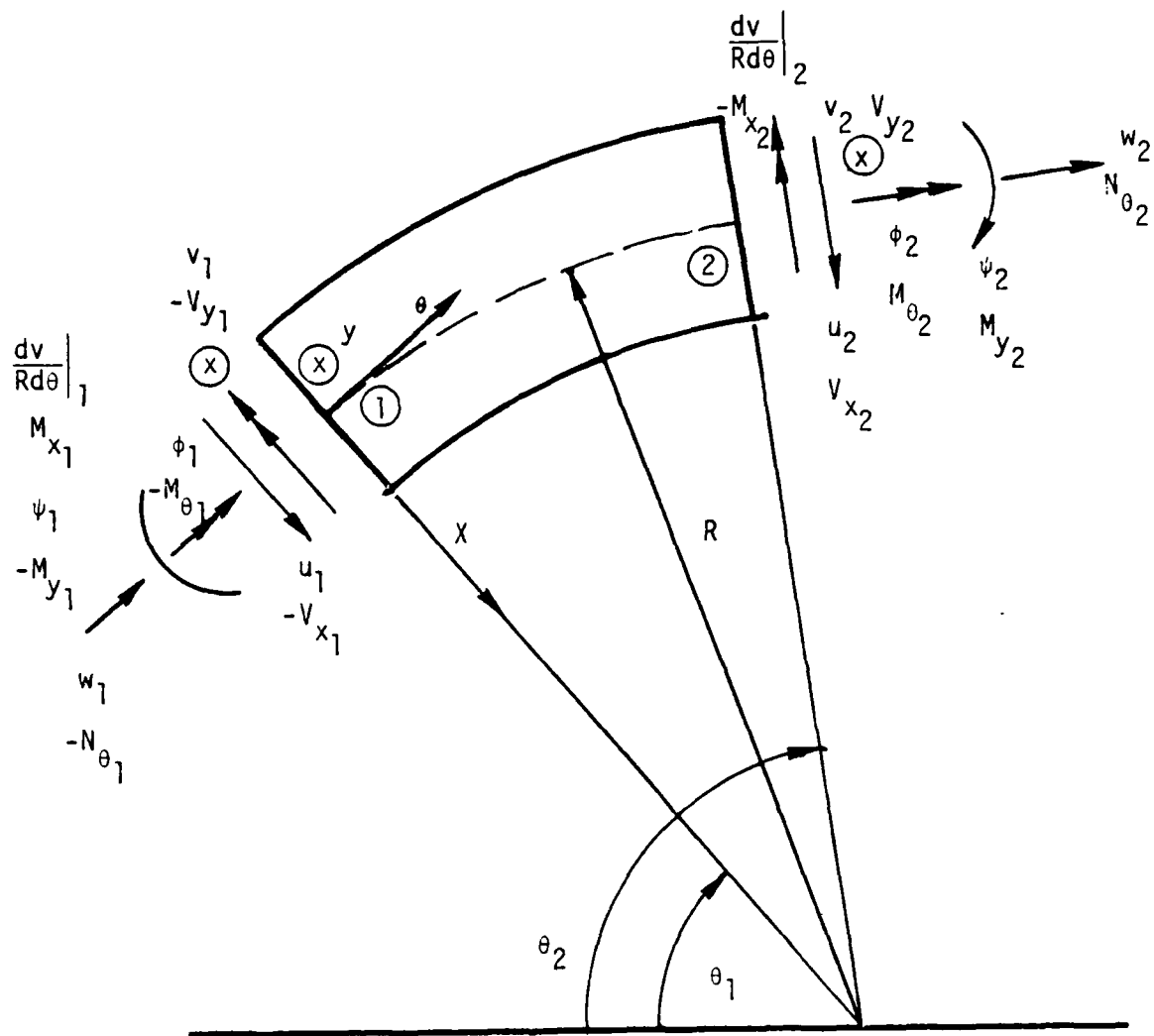


Figure 5-5. Finite Element for Coupled Problem.

Eliminating  $u$  between Equations (5-21) and (5-23)

$$\phi'' + \phi'''' = -v'' - v'''' . \quad (5-69)$$

Substituting for  $v$ ,

$$\phi'' + \phi'''' = -2G_6 \sin \theta + 2G_5 \cos \theta . \quad (5-70)$$

The homogeneous and particular solutions to this equation give

$$\begin{aligned} \phi R = G_7 + G_{15} \theta + G_8 \sin \theta + G_9 \cos \theta \\ - G_5 \theta \sin \theta - G_6 \theta \cos \theta . \end{aligned} \quad (5-71)$$

Rewriting Equation (5-23),

$$(u + u'') \frac{J_{xy}}{J_x} = \phi R - v'' - (\phi'' R + v'') \frac{1}{A} , \quad (5-72)$$

where

$$A = \frac{EJ_x}{GJ_\theta} \quad (5-73)$$

Substituting for  $\phi$  and  $v$ , the equation becomes

$$\begin{aligned} (u + u'') \frac{J_{xy}}{J_x} = G_7 + G_{15} \theta + [(G_8 + G_3) (1 + \frac{1}{A}) + 2G_6] \sin \theta \\ + [(G_9 + G_4) (1 + \frac{1}{A}) - 2G_5] \cos \theta . \end{aligned} \quad (5-74)$$

The homogeneous and particular solutions for  $u$  give



$$\begin{aligned}
\frac{J_{xy}}{J_x} u &= G_7 + G_{15} \theta + G_{10} \sin \theta + G_{11} \cos \theta \\
&+ \left[ \frac{1}{2} (G_9 + G_4) \left( 1 + \frac{1}{A} \right) - G_5 \right] \theta \sin \theta \\
&- \left[ \frac{1}{2} (G_8 + G_3) \left( 1 + \frac{1}{A} \right) + G_6 \right] \theta \cos \theta . \quad (5-75)
\end{aligned}$$

Substitution for  $v$ ,  $\phi$ , and  $u$  into Equation (5-22) shows that

$$G_{15} \left( 1 - \frac{J_{xy}^2}{J_x J_y} \right) = 0 . \quad (5-76)$$

So  $G_{15}$  is an extra constant and

$$G_{15} = 0 . \quad (5-77)$$

Now substituting for  $u$ ,  $v$ , and  $\phi$  into Equation (5-20),

$$\begin{aligned}
w' + w'''' &= \left[ \left( 1 + \frac{J_y}{aR^2} \right) \frac{J_x}{J_{xy}} - \frac{J_{xy}}{aR^2} \right] G_7 \\
&+ 2 \frac{J_x}{J_{xy}} \left[ \frac{1}{2} (G_8 + G_3) \left( 1 + \frac{1}{A} \right) + G_6 \right] \sin \theta \\
&+ 2 \frac{J_x}{J_{xy}} \left[ \frac{1}{2} (G_9 + G_4) \left( 1 + \frac{1}{A} \right) - G_5 \right] \cos \theta . \quad (5-78)
\end{aligned}$$

The particular and homogeneous solutions for  $w$  give

$$\begin{aligned}
w = & G_{12} + \left[ \left( 1 + \frac{J_y}{aR^2} \right) \frac{J_x}{J_{xy}} \right] - \frac{J_{xy}}{aR^2} G_7 + G_{13} \sin \theta + G_{14} \cos \theta \\
& - \frac{J_x}{J_{xy}} \left[ \frac{1}{2} (G_8 + G_3) \left( 1 + \frac{1}{A} \right) + G_6 \right] \theta \sin \theta \\
& - \frac{J_x}{J_{xy}} \left[ \frac{1}{2} (G_9 + G_4) \left( 1 + \frac{1}{A} \right) - G_5 \right] \theta \cos \theta . \quad (5-79)
\end{aligned}$$

We now have the solution for  $u$ ,  $v$ ,  $w$ , and  $\phi$  in terms of 14 arbitrary constants, two more than dictated by the boundary conditions of the problem. To find the extra constants, one must check to see that the original differential equations of equilibrium (5-1) through (5-6) are satisfied. A combination of (5-3) and (5-5) give

$$M'_y + RN'_\theta = 0 . \quad (5-80)$$

But from (5-11)

$$M'_y + RN'_\theta = Ea[w'' - u'] \quad (5-81)$$

so

$$w'' - u' = 0 . \quad (5-82)$$

Substituting for  $w$  and  $u$  from Equations (5-79) and (5-75) into the above shows that  $G_{13}$  and  $G_{14}$  are extra constants and can be expressed in terms of the other constants.

$$G_{13} = \frac{J_x}{J_{xy}} \left[ G_{11} + \frac{1}{2} (G_9 + G_4) \left( 1 + \frac{1}{A} \right) - G_5 \right] , \quad (5-83)$$

$$G_{14} = \frac{J_x}{J_{xy}} \left[ -G_{10} - \frac{1}{2} (G_8 + G_3) \left( 1 + \frac{1}{A} \right) - G_6 \right] . \quad (5-84)$$

Thus, in summary, in terms of the 12 remaining constants,

$$v = G_1 + G_2 \theta + G_3 \sin \theta + G_4 \cos \theta + G_5 \theta \sin \theta + G_6 \theta \cos \theta \quad (5-85)$$

$$\phi = \frac{1}{R} \{G_7 + G_8 \sin \theta + G_9 \cos \theta - G_5 \theta \sin \theta - G_6 \theta \cos \theta\} \quad (5-86)$$

$$u = \frac{J_x}{J_{xy}} \{G_7 + G_{10} \sin \theta + G_{11} \cos \theta + [\frac{1}{2} (G_9 + G_4) (1 + \frac{1}{A}) - G_5] \theta \sin \theta - [\frac{1}{2} (G_8 + G_3) (1 + \frac{1}{A}) + G_6] \theta \cos \theta\} \quad (5-87)$$

$$w = G_{12} + \left[ \left( 1 + \frac{J_y}{aR^2} \right) \frac{J_x}{J_{xy}} - \frac{J_{xy}}{aR^2} \right] G_7 \theta + \frac{J_x}{J_{xy}} \left\{ [G_{11} + \frac{1}{2} (G_9 + G_4) (1 + \frac{1}{A}) - G_5] \sin \theta + [-G_{10} - \frac{1}{2} (G_8 + G_3) (1 + \frac{1}{A}) - G_6] \cos \theta - [\frac{1}{2} (G_8 + G_3) (1 + \frac{1}{A}) + G_6] \theta \sin \theta - [\frac{1}{2} (G_9 + G_4) (1 + \frac{1}{A}) - G_5] \theta \cos \theta \right\}. \quad (5-88)$$

To satisfy the boundary conditions one needs in addition

$$\begin{aligned} \psi = \frac{u'}{R} + \frac{w}{R} = \frac{1}{R} \left\{ G_{12} + \left[ \left( 1 + \frac{J_y}{aR^2} \right) \frac{J_x}{J_{xy}} - \frac{J_{xy}}{aR^2} \right] G_7 \theta \right\} \\ + \frac{1}{R} \frac{J_x}{J_{xy}} \left\{ 2 \left[ \frac{1}{2} (G_9 + G_4) \left( 1 + \frac{1}{A} \right) - G_5 \right] \sin \theta \right. \\ \left. - 2 \left[ \frac{1}{2} (G_8 + G_3) \left( 1 + \frac{1}{A} \right) + G_6 \right] \cos \theta \right\}, \quad (5-89) \end{aligned}$$

$$\begin{aligned} \frac{v'}{R} = \frac{1}{R} \left\{ G_2 + G_3 \cos \theta - G_4 \sin \theta + G_5 (\theta \cos \theta + \sin \theta) \right. \\ \left. + G_6 (-\theta \sin \theta + \cos \theta) \right\}, \quad (5-90) \end{aligned}$$

Applying the boundary conditions shown in Figure 5-4 gives the matrix Equation (5-91) (next page) which is equivalent to

$$[A_{\text{coupled}}] [G] = [\delta_{\text{coupled}}] \quad (5-92)$$

To get the corresponding forces, substituting for  $u$ ,  $v$ ,  $\phi$ , and  $w$  into-Equations (5-11) through (5-14) and (5-3) and (5-4) gives

$$\begin{aligned} V_x = - \frac{EJ_x}{R^3} \left\{ \left( \frac{J_y}{J_{xy}} - \frac{J_{xy}}{J_x} \right) (2G_6 \cos \theta + 2G_5 \sin \theta) \right. \\ \left. + \left[ \frac{J_y}{J_{xy}} \left( 1 + \frac{1}{A} \right) - \frac{J_{xy}}{J_x} \right] \left[ - (G_9 + G_4) \sin \theta + (G_8 + G_3) \cos \theta \right] \right\}, \quad (5-93) \end{aligned}$$

$$V_y = \frac{EJ_x}{R^3} \frac{G_2}{A}, \quad (5-94)$$

0	0	$-Q \theta_1 \cos \theta_1$	$Q \theta_1 \sin \theta_1$	$-P \theta_1 \sin \theta_1$	$-P \theta_1 \cos \theta_1$	$P$	$-Q \theta_1 \cos \theta_1$	$Q \theta_1 \sin \theta_1$	$P \sin \theta_1$	$P \cos \theta_1$	0	$g_1$	$u_1$
1	$\theta_1$	$\sin \theta_1$	$\cos \theta_1$	$\theta_1 \sin \theta_1$	$\theta_1 \cos \theta_1$	0	0	0	0	0	0	$g_2$	$v_1$
0	0	$-Q(\cos \theta_1 + \theta_1 \sin \theta_1)$	$Q(\sin \theta_1 - \theta_1 \cos \theta_1)$	$-P(\sin \theta_1 - \theta_1 \cos \theta_1)$	$-P(\cos \theta_1 + \theta_1 \sin \theta_1)$	$S \theta_1$	$-Q(\cos \theta_1 + \theta_1 \sin \theta_1)$	$Q(\sin \theta_1 - \theta_1 \cos \theta_1)$	$-P \cos \theta_1$	$P \sin \theta_1$	1	$g_3$	$w_1$
0	$\frac{1}{R}$	$\frac{1}{R} \cos \theta_1$	$-\frac{1}{R} \sin \theta_1$	$\frac{1}{R}(\theta_1 \cos \theta_1 + \sin \theta_1)$	$\frac{1}{R}(-\theta_1 \sin \theta_1 + \cos \theta_1)$	0	0	0	0	0	0	$g_4$	$\frac{1}{R} \frac{dv}{d\theta} \Big _1$
0	0	$-\frac{2}{R} Q \cos \theta_1$	$\frac{2}{R} Q \sin \theta_1$	$-\frac{2}{R} P \sin \theta_1$	$-\frac{2}{R} P \cos \theta_1$	$\frac{S}{R} \theta_1$	$-\frac{2}{R} Q \cos \theta_1$	$\frac{2}{R} Q \sin \theta_1$	0	0	$\frac{1}{R}$	$g_5$	$\psi_1$
0	0	0	0	$-\frac{1}{R} \theta_1 \sin \theta_1$	$-\frac{1}{R} \theta_1 \cos \theta_1$	$\frac{1}{R}$	$\frac{1}{R} \sin \theta_1$	$\frac{1}{R} \cos \theta_1$	0	0	0	$g_6$	$\phi_1$
0	0	$-Q \theta_2 \cos \theta_2$	$Q \theta_2 \sin \theta_2$	$-P \theta_2 \sin \theta_2$	$-P \theta_2 \cos \theta_2$	$P$	$-Q \theta_2 \cos \theta_2$	$Q \theta_2 \sin \theta_2$	$P \sin \theta_2$	$P \cos \theta_2$	0	$g_7$	$u_2$
1	$\theta_2$	$\sin \theta_2$	$\cos \theta_2$	$\theta_2 \sin \theta_2$	$\theta_2 \cos \theta_2$	0	0	0	0	0	0	$g_8$	$v_2$
0	0	$-Q(\cos \theta_2 + \theta_2 \sin \theta_2)$	$Q(\sin \theta_2 - \theta_2 \cos \theta_2)$	$-P(\sin \theta_2 - \theta_2 \cos \theta_2)$	$-P(\cos \theta_2 + \theta_2 \sin \theta_2)$	$S \theta_2$	$-Q(\cos \theta_2 + \theta_2 \sin \theta_2)$	$Q(\sin \theta_2 - \theta_2 \cos \theta_2)$	$-P \cos \theta_2$	$P \sin \theta_2$	1	$g_9$	$w_2$
0	$\frac{1}{R}$	$\frac{1}{R} \cos \theta_2$	$-\frac{1}{R} \sin \theta_2$	$\frac{1}{R}(\theta_2 \cos \theta_2 + \sin \theta_2)$	$\frac{1}{R}(-\theta_2 \sin \theta_2 + \cos \theta_2)$	0	0	0	0	0	0	$g_{10}$	$\frac{1}{R} \frac{dv}{d\theta} \Big _2$
0	0	$-\frac{2}{R} Q \cos \theta_2$	$\frac{2}{R} Q \sin \theta_2$	$-\frac{2}{R} P \sin \theta_2$	$-\frac{2}{R} P \cos \theta_2$	$\frac{S}{R} \theta_2$	$-\frac{2}{R} Q \cos \theta_2$	$\frac{2}{R} Q \sin \theta_2$	0	0	$\frac{1}{R}$	$g_{11}$	$\psi_2$
0	0	0	0	$-\frac{1}{R} \theta_2 \sin \theta_2$	$-\frac{1}{R} \theta_2 \cos \theta_2$	$\frac{1}{R}$	$\frac{1}{R} \sin \theta_2$	$\frac{1}{R} \cos \theta_2$	0	0	0	$g_{12}$	$\phi_2$

=

$$P = \frac{J_x}{J_{xy}}, \quad Q = \frac{1}{2} \frac{J_x}{J_{xy}} \left(1 + \frac{1}{A}\right), \quad S = \left(1 + \frac{J_x}{AR^2}\right) \frac{J_{xy}}{J_{xy}} - \frac{J_{xy}}{AR^2}$$

Equation (5-91)

$$N_{\theta} = -\frac{EJ_x}{R^3} \left\{ \left( \frac{J_y}{J_{xy}} - \frac{J_{xy}}{J_x} \right) (2G_6 \sin \theta - 2G_5 \cos \theta) + \left[ \frac{J_y}{J_{xy}} \left( 1 + \frac{1}{A} \right) - \frac{J_{xy}}{J_x} \right] [(G_9 + G_4) \cos \theta + (G_8 + G_3) \sin \theta] \right\}, \quad (5-95)$$

$$M_x = -\frac{EJ_x}{R^2} \frac{1}{A} [(G_3 + G_8) \sin \theta + (G_4 + G_9) \cos \theta], \quad (5-96)$$

$$M_y = \frac{EJ_x}{R^2} \left\{ \left( \frac{J_y}{J_{xy}} - \frac{J_{xy}}{J_x} \right) (G_7 - 2G_5 \cos \theta + 2G_6 \sin \theta) + \left[ \frac{J_y}{J_{xy}} \left( 1 + \frac{1}{A} \right) - \frac{J_{xy}}{J_x} \right] [(G_9 + G_4) \cos \theta + (G_8 + G_3) \sin \theta] \right\}. \quad (5-97)$$

$$M_{\theta} = \frac{EJ_x}{R^2} \frac{1}{A} [G_2 + (G_3 + G_8) \cos \theta - (G_4 + G_9) \sin \theta]. \quad (5-98)$$

Using the above, the generalized force vector is given by Equation (5-99) (next page). This equation has the form

$$\frac{EJ_x}{R^2} [D_{\text{coupled}}] [G] = [F_{\text{coupled}}]. \quad (5-100)$$

Using Equations (5-92) and (5-100)

$$[F_{\text{coupled}}] = \frac{EJ_x}{R^2} [D_{\text{coupled}}] [A_{\text{coupled}}]^{-1} [\delta_{\text{coupled}}]. \quad (5-101)$$

Thus,

$$\frac{EJ_x}{R^2}$$

$$\begin{bmatrix}
 0 & 0 & \frac{V}{R} \cos \theta_1 & -\frac{V}{R} \sin \theta_1 & \frac{2U}{R} \sin \theta_1 & \frac{2U}{R} \cos \theta_1 & 0 & \frac{V}{R} \cos \theta_1 & -\frac{V}{R} \sin \theta_1 & 0 & 0 & 0 \\
 0 & -\frac{1}{RA} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & \frac{V}{R} \sin \theta_1 & \frac{V}{R} \cos \theta_1 & -\frac{2U}{R} \cos \theta_1 & \frac{2U}{R} \sin \theta_1 & 0 & \frac{V}{R} \sin \theta_1 & \frac{V}{R} \cos \theta_1 & 0 & 0 & 0 \\
 0 & 0 & -\frac{1}{A} \sin \theta_1 & -\frac{1}{A} \cos \theta_1 & 0 & 0 & 0 & -\frac{1}{A} \sin \theta_1 & -\frac{1}{A} \cos \theta_1 & 0 & 0 & 0 \\
 0 & 0 & -V \sin \theta_1 & -V \cos \theta_1 & 2U \cos \theta_1 & -2U \sin \theta_1 & -U & -V \sin \theta_1 & -V \cos \theta_1 & 0 & 0 & 0 \\
 0 & -\frac{1}{A} & -\frac{1}{A} \cos \theta_1 & \frac{1}{A} \sin \theta_1 & 0 & 0 & 0 & -\frac{1}{A} \cos \theta_1 & \frac{1}{A} \sin \theta_1 & 0 & 0 & 0 \\
 0 & 0 & -\frac{V}{R} \cos \theta_2 & \frac{V}{R} \sin \theta_2 & -\frac{2U}{R} \cos \theta_2 & -\frac{2U}{R} \sin \theta_2 & 0 & -\frac{V}{R} \cos \theta_2 & \frac{V}{R} \sin \theta_2 & 0 & 0 & 0 \\
 0 & \frac{1}{RA} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & -\frac{V}{R} \sin \theta_2 & -\frac{V}{R} \cos \theta_2 & \frac{2U}{R} \cos \theta_2 & -\frac{2U}{R} \sin \theta_2 & 0 & -\frac{V}{R} \sin \theta_2 & -\frac{V}{R} \cos \theta_2 & 0 & 0 & 0 \\
 0 & 0 & \frac{1}{A} \sin \theta_2 & \frac{1}{A} \cos \theta_2 & 0 & 0 & 0 & \frac{1}{A} \sin \theta_2 & \frac{1}{A} \cos \theta_2 & 0 & 0 & 0 \\
 0 & 0 & V \sin \theta_2 & V \cos \theta_2 & -2U \cos \theta_2 & 2U \sin \theta_2 & U & V \sin \theta_2 & V \cos \theta_2 & 0 & 0 & 0 \\
 0 & \frac{1}{A} & \frac{1}{A} \cos \theta_2 & -\frac{1}{A} \sin \theta_2 & 0 & 0 & 0 & \frac{1}{A} \cos \theta_2 & -\frac{1}{A} \sin \theta_2 & 0 & 0 & 0
 \end{bmatrix}
 =
 \begin{bmatrix}
 G_1 \\
 G_2 \\
 G_3 \\
 G_4 \\
 G_5 \\
 G_6 \\
 G_7 \\
 G_8 \\
 G_9 \\
 G_{10} \\
 G_{11} \\
 G_{12}
 \end{bmatrix}
 =
 \begin{bmatrix}
 -V_{x1} \\
 -V_{y1} \\
 -M_{\theta 1} \\
 M_{x1} \\
 -M_{y1} \\
 -M_{\theta 1} \\
 V_{x2} \\
 V_{y2} \\
 M_{\theta 2} \\
 -M_{x2} \\
 M_{y2} \\
 M_{\theta 2}
 \end{bmatrix}$$

$$U = \frac{J_y}{J_{xy}} - \frac{J_{xy}}{J_x}, \quad V = \frac{J_y}{J_{xy}} \left(1 + \frac{1}{A}\right) - \frac{J_{xy}}{J_x}$$

Equation (5-99)

$$[K_{\text{coupled}}] = \frac{EJ_x}{R^2} [D_{\text{coupled}}] [A_{\text{coupled}}]^{-1} \quad (5-102)$$

As an example, let

$$R = 3.0$$

$$\theta_1 = 0$$

$$\theta_2 = \pi/6$$

$$A = 5$$

(5-103)

$$\frac{J_y}{aR^2} = 0.01$$

$$\frac{J_{xy}}{J_x} = 0.1$$

$$\frac{J_{xy}}{J_y} = 0.1$$

For this case the stiffness matrix  $[K_{\text{coupled}}]$  is calculated as

$$[K_{\text{coupled}}] = \frac{EJ_x}{R^2} \begin{bmatrix} 30.71 & 2.64 & 9.21 & 2.03 & 21.84 & 0.17 & -21.99 & -2.64 & -23.33 & 2.12 & 20.53 & -0.19 \\ 2.64 & 27.12 & -0.71 & 21.20 & 2.12 & 2.24 & -2.64 & -27.12 & -0.71 & 21.20 & 2.12 & -2.24 \\ 9.21 & -0.71 & 62.62 & -0.72 & -3.24 & -0.09 & 23.33 & 0.71 & -58.83 & -0.39 & -8.11 & 0.00 \\ 2.03 & 21.20 & -0.72 & 21.55 & 2.16 & 2.77 & -2.12 & -21.20 & -0.39 & 11.75 & 1.17 & -0.15 \\ 21.84 & 2.12 & -3.24 & 2.16 & 22.79 & 0.28 & -20.53 & -2.12 & -8.11 & 1.17 & 11.26 & -0.01 \\ 0.17 & 2.24 & -0.09 & 2.77 & 0.28 & 1.63 & -0.19 & -2.24 & -0.00 & 0.15 & 0.01 & -0.92 \\ -21.99 & -2.64 & 23.33 & -2.12 & -20.53 & -0.19 & 30.71 & 2.64 & -9.21 & -2.03 & -21.84 & 0.17 \\ -2.64 & -27.12 & 0.71 & -21.20 & -2.12 & -2.24 & 2.64 & 27.12 & 0.71 & -21.20 & -2.12 & 2.24 \\ -23.33 & -0.71 & -58.83 & -0.39 & -8.11 & -0.00 & -9.21 & 0.71 & 62.62 & -0.72 & -3.24 & 0.09 \\ 2.12 & 21.20 & -0.39 & 11.75 & 1.17 & 0.15 & -2.03 & -21.20 & -0.72 & 21.55 & 2.16 & -2.77 \\ 20.53 & 2.12 & -8.11 & 1.17 & 11.26 & 0.01 & -21.84 & -2.12 & -3.24 & 2.16 & 22.79 & -0.28 \\ -0.19 & -2.24 & 0.00 & -0.15 & -0.01 & -0.92 & 0.17 & 2.24 & 0.09 & -2.77 & -0.28 & 1.63 \end{bmatrix}$$

(5-104)



Verification of Stiffness Matrices--The correctness of the preceding stiffness matrices has been verified in detail by Knowlton [24]. He assembled the elements into rings, performed solutions and compared to known results. The in-plane results were compared to those of Morris [49] and straight beam theory. There is some slight difference when compared to Morris because he uses straight beam theory. Comparison is perfect of course as  $R \rightarrow \infty$ .

The out-of-plane case was verified by comparing to a problem solution for out-of-plane deflection presented by Roark [44]. Agreement was perfect.

Finally for the coupled problem, the only completely independent solutions formed are those by Meck [47]. Comparison was made to a cantilever curved beam section loaded in two planes and with  $J_{xy} \neq 0$ . Again, agreement was perfect. Coupled numerical solutions were also compared to those analytical solutions presented by Lebeck [41]. Again agreement was perfect.

Thus, based on these limited checks the preceding results appear to be correct. The reader is again cautioned that warping is not included here and this is valid for compact sections. However, warping does play a major role in many practical problems and must be included where necessary.

#### General Contact Problem

Given the stiffness matrix results for a ring, the next step in the solution of the general seal ring contact problem is to create a model of contact and incorporate it into problem solutions. Before illustrating how this is to be done for the actual seal assembly, a simple example will be used to show how such a contact problem can be solved.

The example problem will start with a beam on an elastic foundation (Figure 5-6). This problem can be modeled using beam finite elements (Figure 5-7). The stiffness matrix for a beam element as given by Irons [51] is

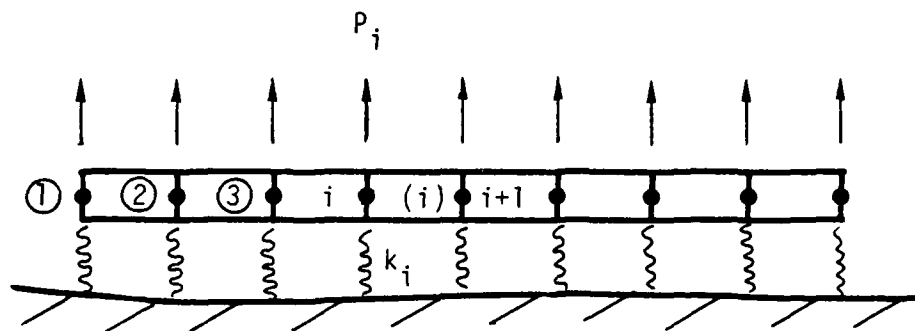


Figure 5-6. Beam on Elastic Foundation.

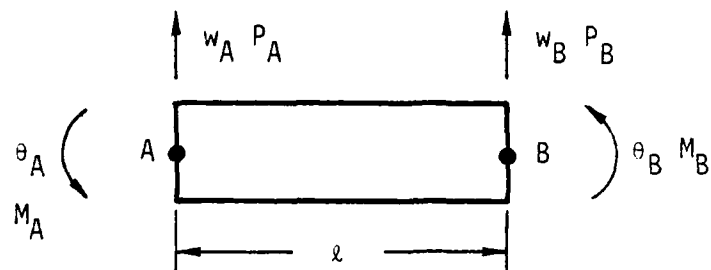


Figure 5-7. Beam Element.

$$K_e = \frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l & -6l & 4l \end{bmatrix} \quad (5-105)$$

where the displacement and force vectors are

$$[\delta] = \begin{bmatrix} w_A \\ \theta_A \\ w_B \\ \theta_B \end{bmatrix} \quad (5-106)$$

$$[F] = \begin{bmatrix} P_A \\ M_A \\ P_B \\ M_B \end{bmatrix} \quad (5-107)$$

For a solution by the finite element method, one requires an expression for the total strain energy of the system. The strain energy for one beam element and the springs under the element is

$$U^{(e)} = \frac{1}{2} [\delta]^T [K_e] [\delta] + \frac{1}{2} \frac{k_A}{2} (w_A - \Delta_{0A})^2 + \frac{1}{2} \frac{k_B}{2} (w_B - \Delta_{0B})^2 . \quad (5-108)$$

The terms  $k_A/2$  and  $k_B/2$  assume the spring under each node is divided into two equal parts, one part associated with each adjacent element.

Total potential energy is the strain energy over all of the elements plus that due to the potential of applied loads or

$$U = \sum_e U^{(e)} - [F]^T [\delta] . \quad (5-109)$$

According to the principle of minimum potential energy

$$\delta U = 0 . \quad (5-110)$$

Applying this principle to Equations (5-108) and (5-109) combined gives

$$\left\{ \sum_e [K_e] + \sum_e \left[ \frac{k_A}{2} (w_A - \Delta_{0A}) + \frac{k_B}{2} (w_B - \Delta_{0A}) \right]_e \right\} [\delta] = \sum_e [F_e] . \quad (5-111)$$

In matrix form, using the numbering system shown in Figure (5-6), the system of equations to be solved is:

$$\frac{EI}{L^3} \begin{bmatrix} 12 + \frac{k_1 L^3}{EI} & 6L & -12 & 6L & & & & & \\ & 6L & 4L^2 & -6L & 2L^2 & & & & \\ & -12 & -6L & 24 + \frac{k_2 L^3}{EI} & 0 & -12 & 6L & & \\ & 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 & & \\ & & & -12 & -6L & 24 + \frac{k_3 L^3}{EI} & 0 & -12 & 6L \\ & & & 6L & 2L^2 & 0 & 8L^2 & -6L & 2L^2 \\ & & & & & -12 & -6L & 24 + \frac{k_4 L^3}{EI} & 0 \\ & & & & & 6L & 2L^2 & 0 & 8L^2 \\ & & & & & & \dots & \dots & \\ & & & & & & \dots & \dots & \end{bmatrix} \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \\ w_4 \\ \theta_4 \\ \vdots \\ \vdots \end{bmatrix} = \begin{bmatrix} P_1 - \frac{k_1 \Delta_1}{2} \\ \\ P_2 - k_2 \Delta_2 \\ \\ P_3 - k_3 \Delta_3 \\ \\ P_4 - k_4 \Delta_4 \\ \vdots \\ \vdots \end{bmatrix} \quad (5-112)$$

System of Equations (5-112) can be readily solved for the equilibrium displacements  $w_i$  for arbitrary  $k_i$  and  $P_i$ . The  $\Delta_i$  allow for any preload of the springs. An algorithm solving Equation (5-112) was written and solutions to various beam on elastic foundation problems were carried out. For the case of a semi-infinite beam on elastic foundation loaded at the end only, the displacement at the end was found to be within 9.2 percent with 20 elements, 2.5 percent with 40 elements, and 0.4 percent with 80 elements of the exact solution results. Thus the method was verified.

Now to solve a contact problem one need only make the springs stiff enough to approximate contact stiffness, allow for preload by adjusting the  $\Delta_i$ , and allow for noncontact by causing the  $k_i \rightarrow 0$  where appropriate. The example problem to be solved is shown in Figure 5-8. Here a beam is brought into contact with two circularly curved surfaces of high rigidity. Loads  $P_1$  and  $P_2$  act to deflect the beam making it contact the two surfaces with some knowable contact pressure distribution. The problem to be solved is to find this pressure distribution, or more precisely to find which points in the discretized problem make contact and at what load.

The procedure used to solve the stated problem is as follows:

- 1) Pick initial contact points A and B and set  $k_A$  and  $k_B$  equal to contact stiffness. Set all other  $k_i \rightarrow 0$ .
- 2) Set  $P_1' = nF \times P_1$  and  $P_2' = nF \times P_2$  where  $F$  is some fraction of 1.0 and  $n = 1$  ( $F = 0.1$  appears to work). Solve for the deflection using this reduced load.
- 3) Compare the deflection to the known gap. If deflection is greater than gap at  $i$ , set  $k_i$  equal to contact stiffness and set  $\Delta_i$  also.
- 4) If deflection is less than  $\Delta$  at  $i$ , the set  $k_i \rightarrow 0$ .
- 5) Repeat steps 2), 3), and 4) with  $n = 2, 3 \dots 1/F$ .
- 6) If assumed contact pattern agrees with solved for contact pattern at  $nF = 1$ , the solution is consistent and in hand. If it does not, then set  $F$  equal to a finer increment at least near full load.

The above procedure was found to give a unique solution to the example problem. It was found that only by slowly increasing the load was it possible to arrive at a consistent solution (see 6) above). Applying full load resulted in continual changes in contact pattern with each iteration--some of the time.

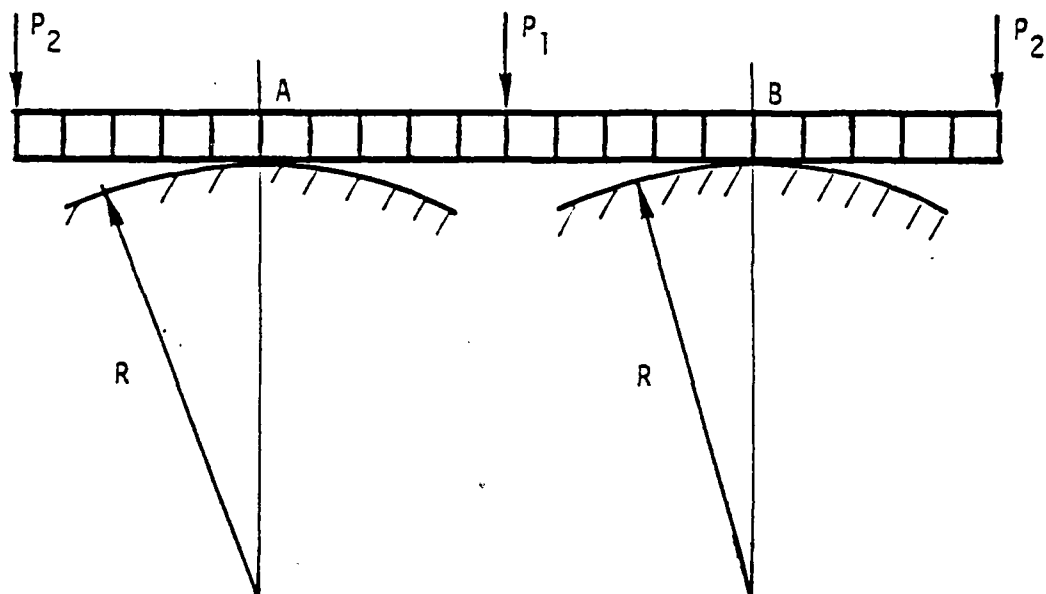


Figure 5-8. Beam Contact Problem.

For the example problem,

$$L = 1 \text{ in. (total length 20 in.)}$$

$$E = 30 \times 10^6 \text{ lb/in.}^2$$

$$k_i = 100 \times 10^6 \text{ lb/in.}$$

$$P_1 = 20000 \text{ lb} \quad (5-113)$$

$$P_2 = 10000 \text{ lb}$$

$$R = 100 \text{ in.}$$

$$I = 0.085 \text{ in.}^4$$

$$\Delta_i - \text{based on circular support}$$

The computed result is shown in Table 5-1.

#### Ring Contact Problem

The contact problem for two rings is considerably more difficult than that for the beam. Figure 5-9 shows how the problem can be set up. The two seal rings are pushed together by arbitrary loads.  $k_s$  represents the stiffness of the support and  $k_f$  is the stiffness of the contact at the face. The contact is represented only by one set of springs at the mean contact radius. Additional sets could be included to better model face contact, but it is thought that for initial analysis at least, one set is sufficient.

The solution to the problem represented in Figure 5-9 is accomplished in much the same way as that for the beam. First, using the stiffness matrices derived for the ring elements, the strain energy of the entire assembly including the springs is found. Then, derivatives are taken and the set of resulting equations solved.

These final steps remain to be completed.

Table 5-1  
Computed Results - Beam Contact Problem

Node	$w_i$	$\Delta_i$	Contact Force
1	-0.1141	-0.1251	0
2	-0.07761	-0.08003	0
3	-0.04505	-0.04501	3459
4	-0.02007	-0.02000	7014
5	-0.005011	-0.005005	602
6	-0.00008922	-0.0	8922
7	-0.004347	-0.005005	0
8	-0.01446	-0.02000	0
9	-0.02652	-0.04501	0
10	-0.03662	-0.08003	0
11	-0.04085	-0.1251	0
12	-0.03662	-0.08003	0
13	-0.02652	-0.04501	0
14	-0.01446	-0.02000	0
15	-0.004347	-0.005005	0
16	-0.00008922	-0.0	8922
17	-0.005011	-0.005005	602
18	-0.02007	-0.02000	7014
19	-0.04505	-0.04501	3459
20	-0.07761	-0.08003	0
21	-0.1141	-0.1251	0



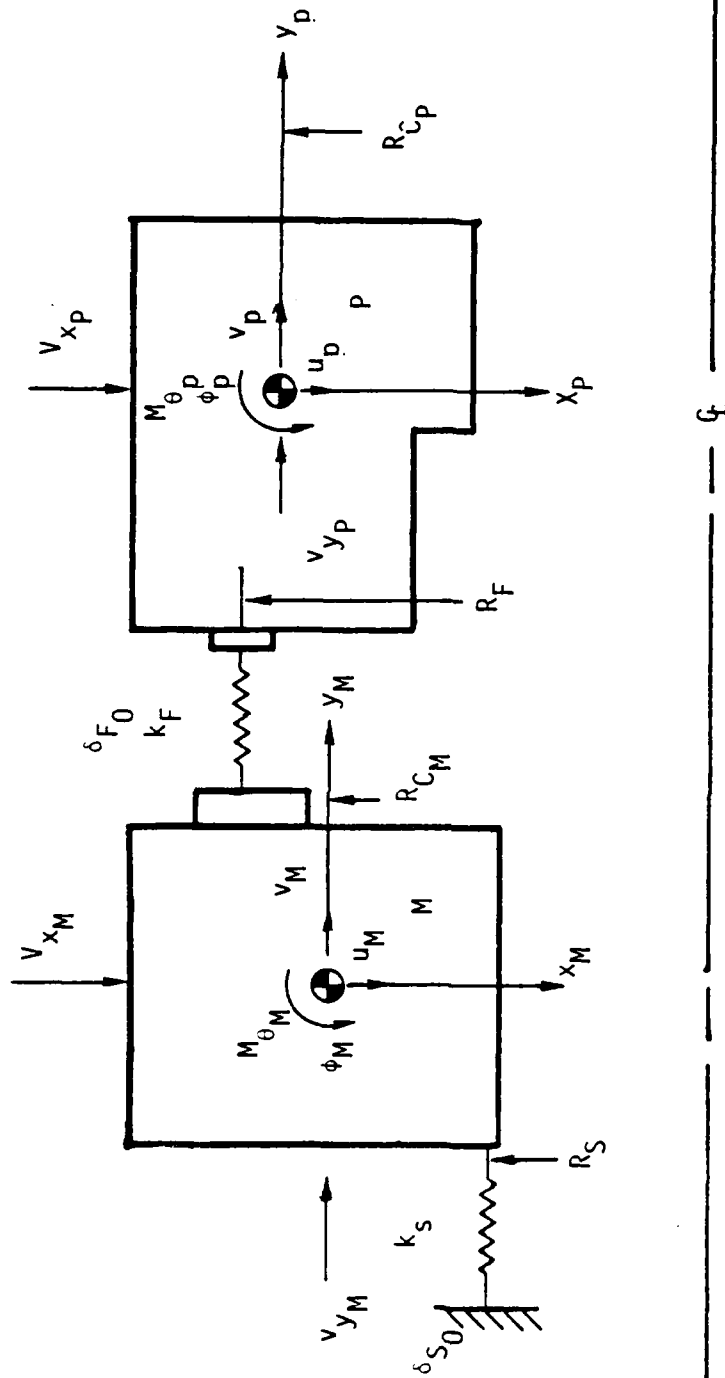


Figure 5-9. Two Ring Contact Problem.

## CHAPTER 6

### SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

#### Nine-Wave Seal Design

As of Test 117, nine-wave seal performance began to approach expected performance and the various design problems appear to have been solved. Conclusions concerning the long-term performance must await the completion of the scheduled 2000 hour test. However, several conclusions can be reached concerning the design itself.

1) As designed, 54 thin arms must be machined out of a solid Inconel 625 piece. While there was success in accomplishing this difficult machining task, a great amount of NC machine time was required. A simpler design or a more easily worked material must be found for practical application of the concept.

2) As designed, many 1/32-in. holes were needed in the Inconel 625. Again, while there was ultimately success in drilling these holes, a great amount of machine time was required. Special techniques had to be developed to remove broken drills. This problem could probably be overcome by using an EDM machine to make these holes. Alternatively, more easily worked material or a design which does not require so many small holes in a hard to work metal is to be preferred.

3) Larger than expected arm deflections led to hydraulic leakage problems. A 3-D stress analysis of the configuration would have shown this problem or a simpler more readily analyzed configuration should be used.

4) Ring stiffness with regard to waviness turned out to be larger than expected because significant torsional warping stresses developed. Again, a 3-D stress analysis would have shown this problem earlier and should have been used. This problem was overcome by reducing the cross section.

5) A method was developed to retain all O-rings on assembly. This should have been incorporated in the original design.

6) A large thickness epoxy bond of the carbon into the Inconel allows the carbon to creep such that it need not conform to the desired waviness. This problem was eliminated by reducing the bond thickness to 0.001 in. or less.

7) To eliminate hydraulic leakage in spite of the large arm deflection, a piston arrangement was used which gives the O-ring a zero gap through which it can extrude. After trying several other approaches it was concluded that if there exists any seal ring gap which must be sealed by the O-ring, the periodic motion of the arms will encourage the O-ring to extrude through the gap--even though the gap size is perfectly acceptable for a static O-ring application.

#### Test Machine Modification

The test machine was modified so that it can automatically run a seal test consisting of variable speed, pressure, and misalignment. The test becomes completely computer controlled. These changes required considerable modification of the test apparatus and its associated electronics. While existing software was modified to run the 2000 hour test, it became clear that a new software package incorporating greater flexibility and refined logic should be written. The computer controlled test cycle works as planned.

#### Seventy-Five Percent Balance Ratio Test

A 500 hour 75 percent balance ratio flat face test was run to try to duplicate and explain some previous data. It was concluded that the seal can start up in one of at least two modes of operation and that the second mode of high friction zero leakage is not stable.

#### Nine-Wave Seal Design Methodology

Chapter 3 points out the design criteria for the nine-wave seal. Many factors were considered including zero moment design,

location of the centroid, low stiffness, and corrosion resistance. A good part of the design was accomplished using a computer program which relates the various geometrical variables to the design criteria. Alternative configurations were evaluated. Finite element and finite difference stress and deflection calculations were made. Performance was predicted using previously developed models. A clear design methodology has been set forth which should be generally applicable to wavy seal design for other purposes.

#### Misalignment

Since submarine seals undergo misalignment as a submarine descends, a condition for the simulated submarine operation test is to create a similar variable misalignment. This was achieved in a unique way for the wavy seal tests. Using the worm gear drive seal support originally used to move waviness and machining angles into the parts, the amount of misalignment can be adjusted as needed. This saves a lot of time in not having to remisalign the machine by hand for each test condition. The misalignment is computer controlled.

#### Moving Wave Gas Seal

The complete analysis of a moving wave gas seal is included in Chapter 4. Certain advantages of a moving wave gas seal compared to other types are pointed out. The analysis uses the alternating direction implicit method to solve the compressible flow equation. Other aspects of the analysis are similar to those already used for liquid seals. Parameter studies are made which show that at low speed operation the fluid pressure left is predominately due to hydrostatic effects. At zero pressure differential, a very smooth surface is required to allow the seal to hydrodynamically lift off. The design theoretically appears to work and offers a gas seal with very low friction and leakage. Experiments are needed to confirm these predictions.

### Predicting Seal Ring Gap

To address the difficult problem of predicting seal ring gap (or zero gap as is desired) when rings are not axisymmetric or when seals have complex out of flatness, two significant developments were made which will lead to the development of a generalized computer program for predicting seal gap.

1) A new ring finite element was developed for the case of coupled in-plane and out-of-plane loading and deflection. The element has been checked. It provides a relatively simple method to solve the contact problem compared to 3-D solid elements.

2) Contact problems for beams otherwise similar to the one of interest have been solved using an iteration method.

After 1) and 2) are combined, the analysis of actual seal rings can be initiated.

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APPENDIX A  
TEST RESULTS



# APPENDIX A

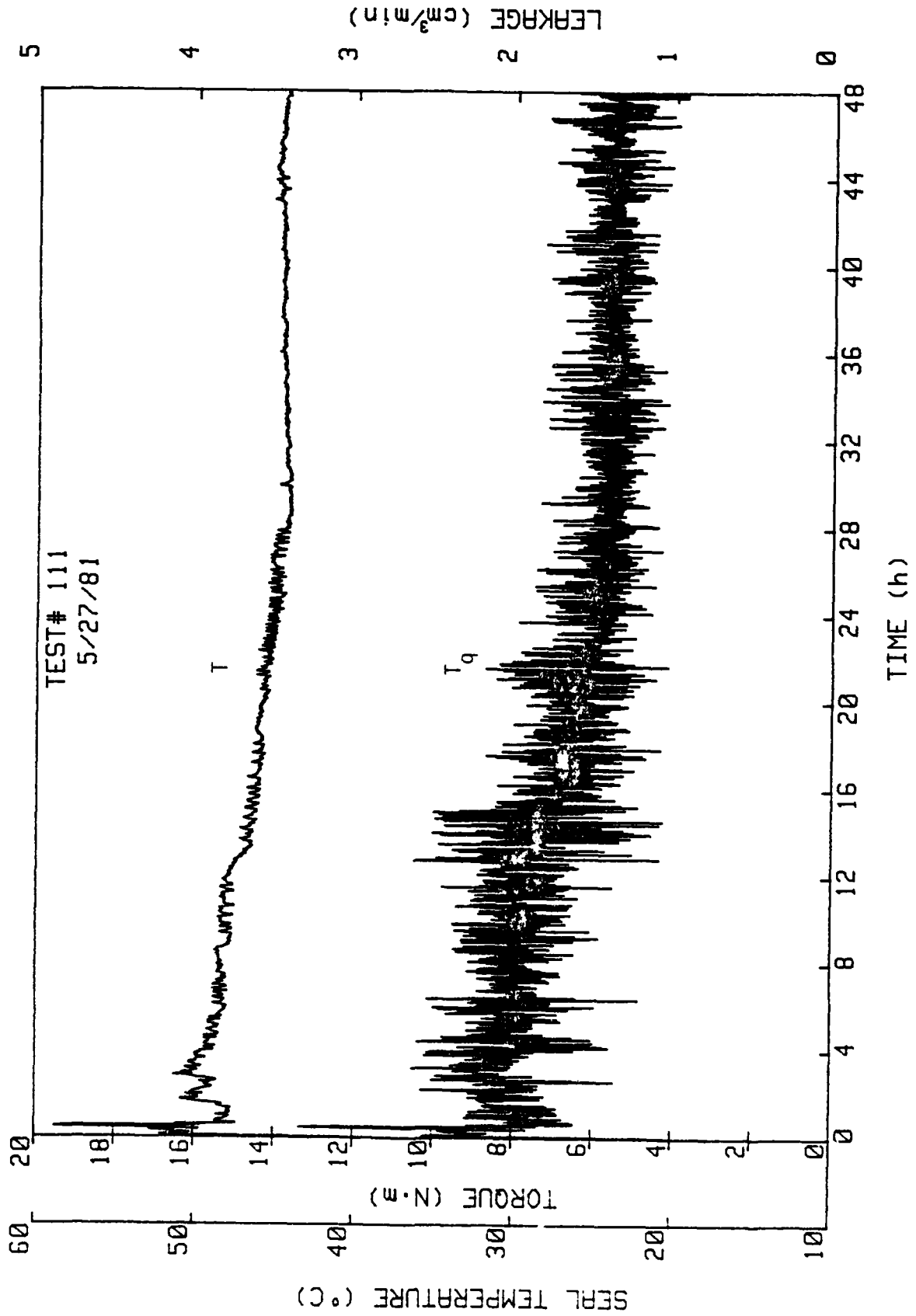
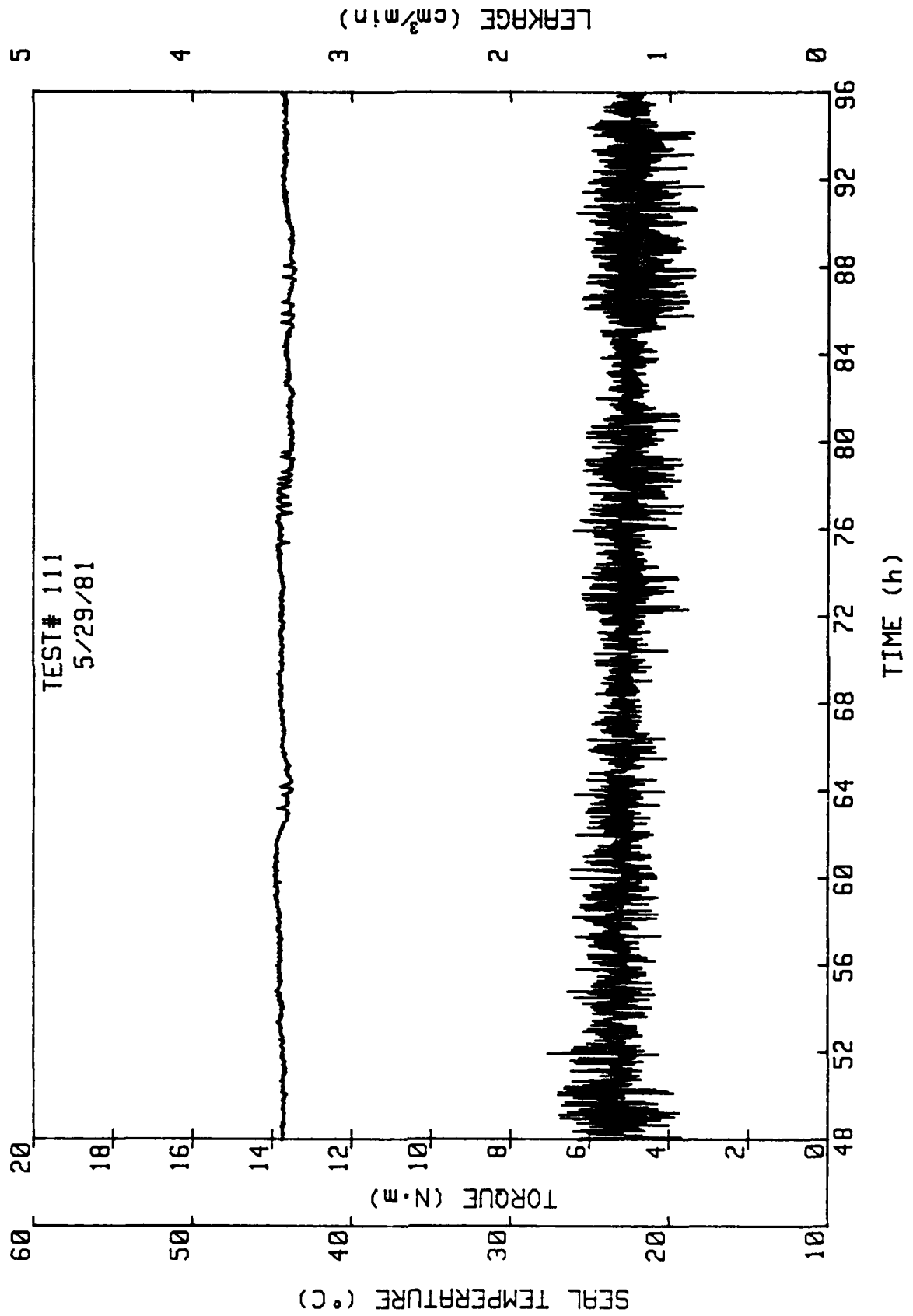
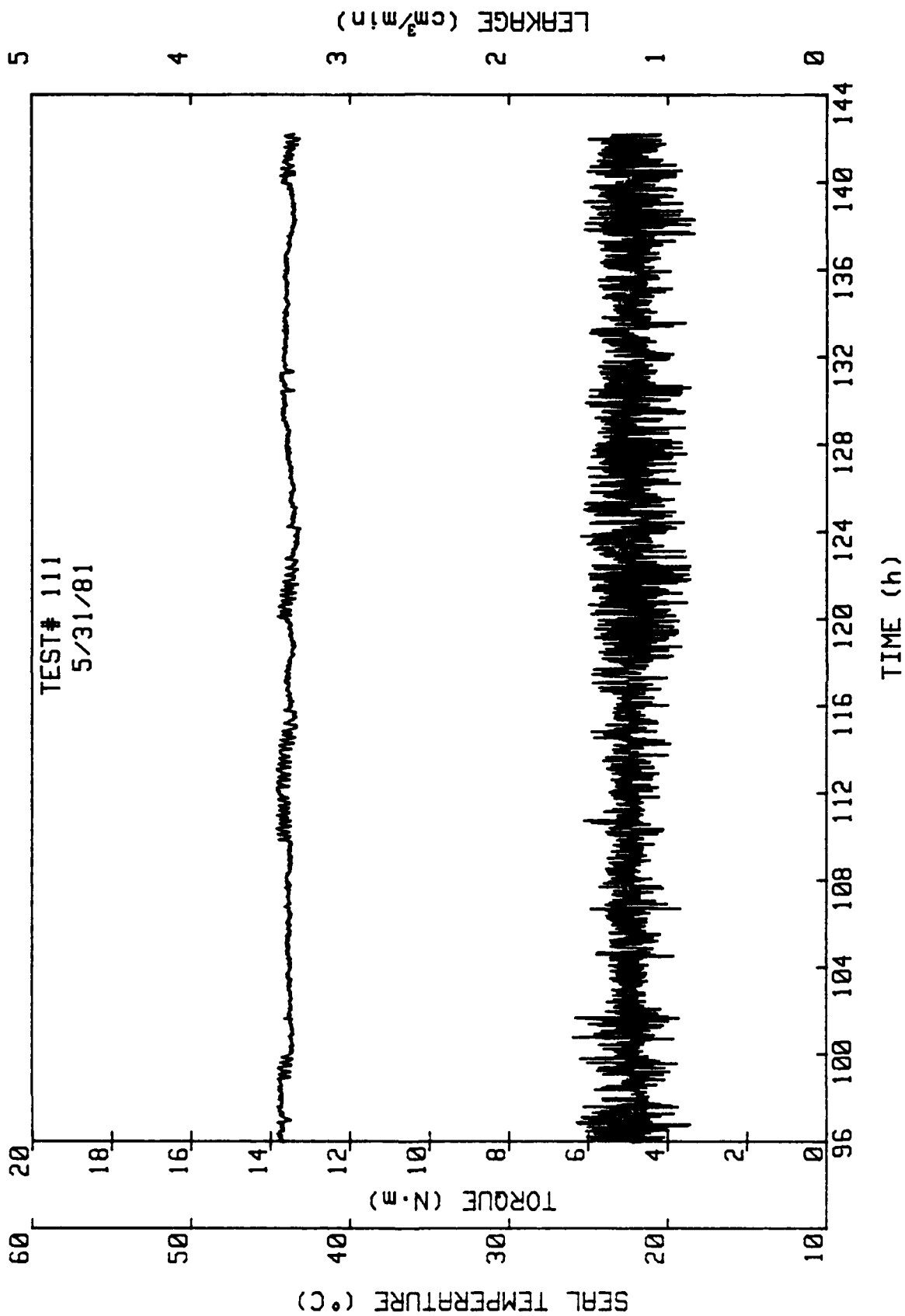


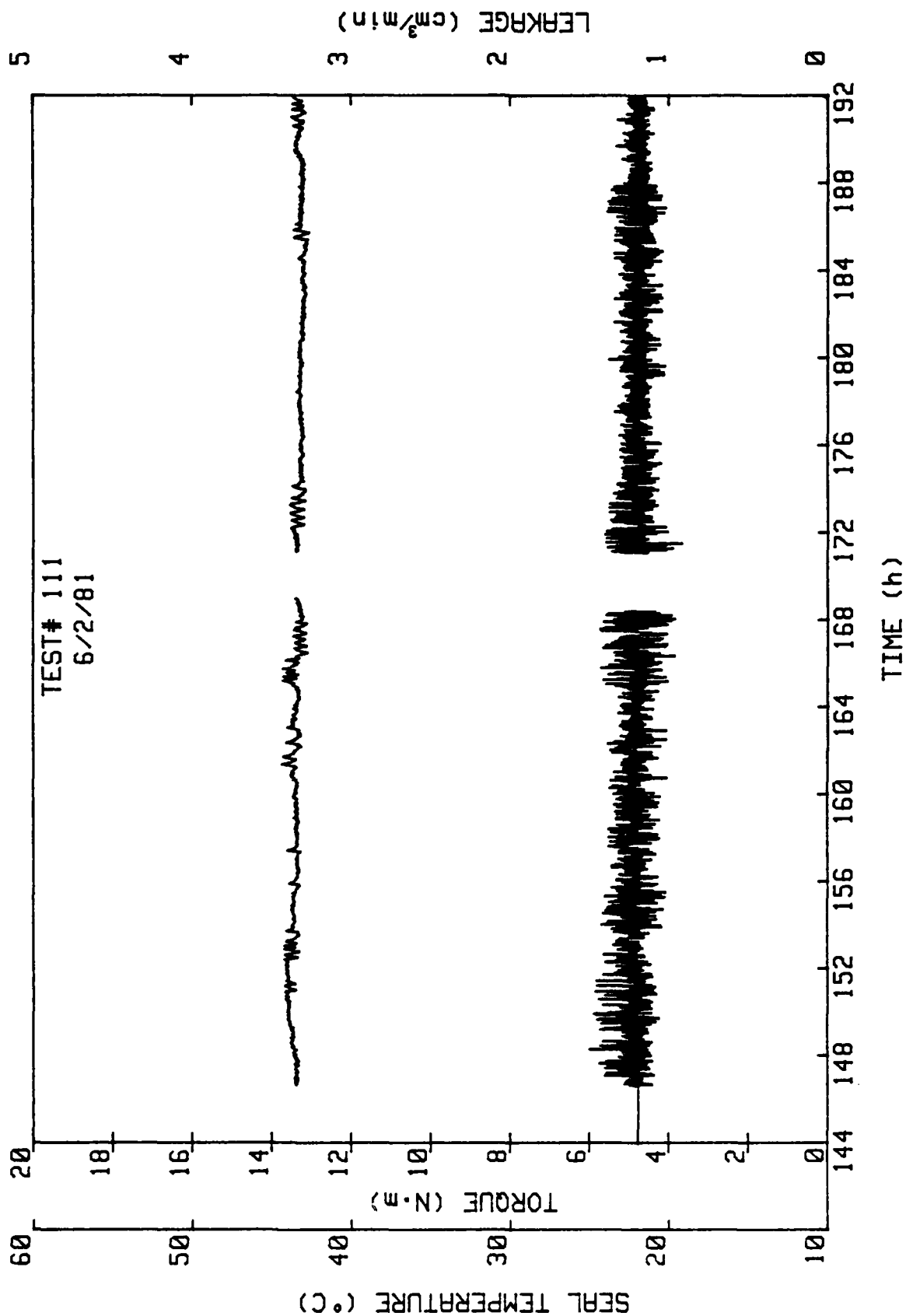
Figure A-1. Test #111, 75% Balance, Parallel Faces, 500 psi, 1800 rpm.

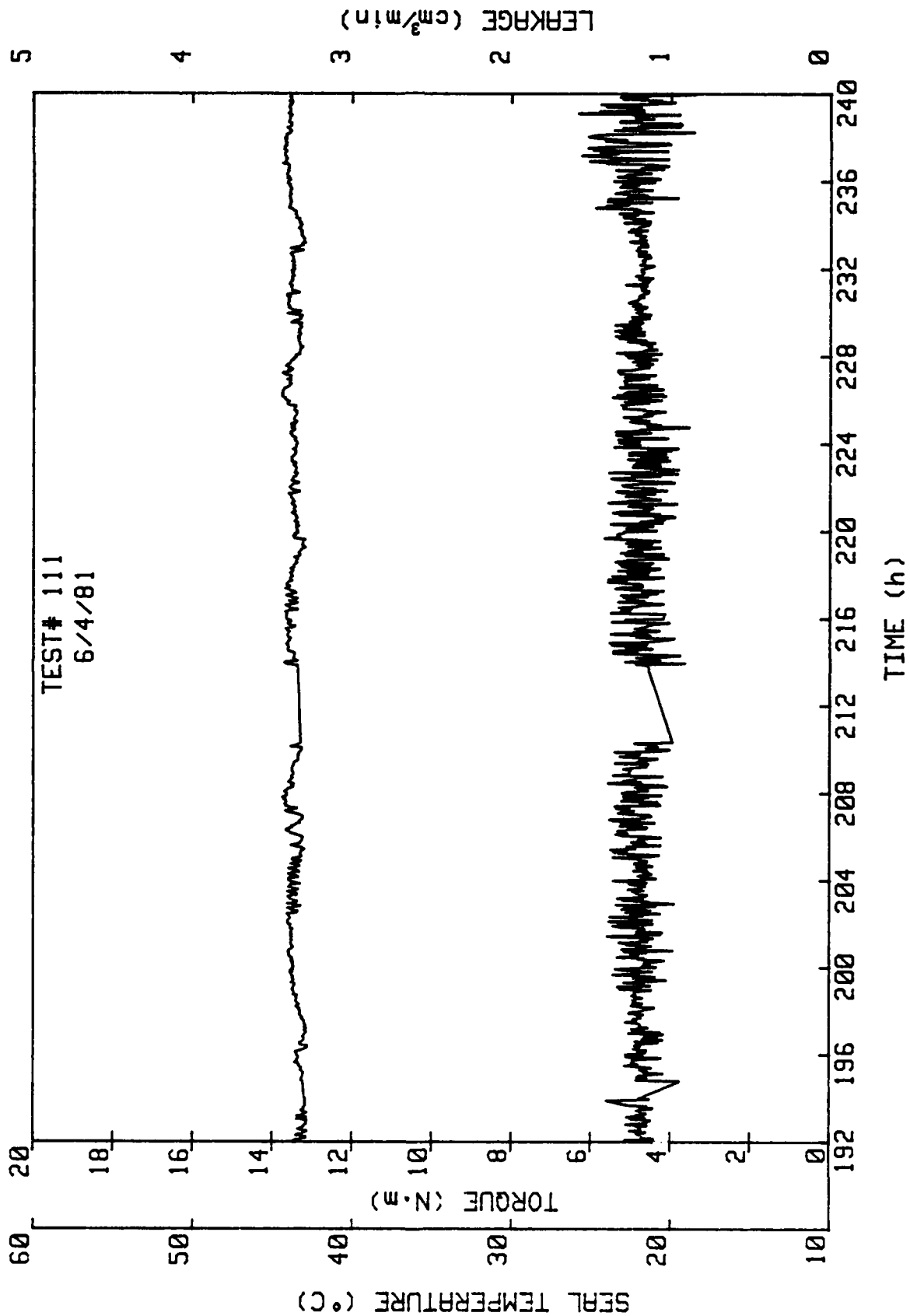
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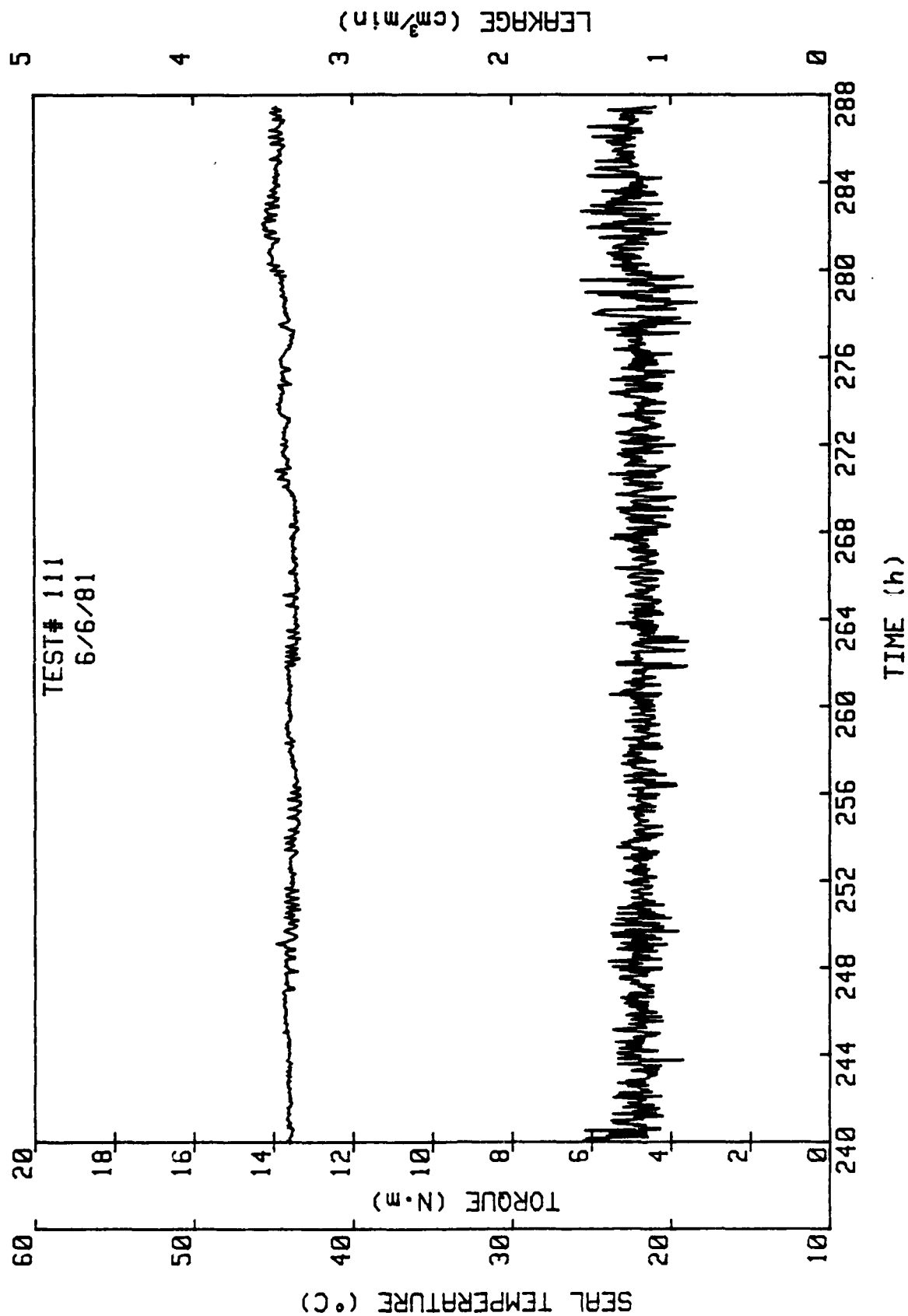


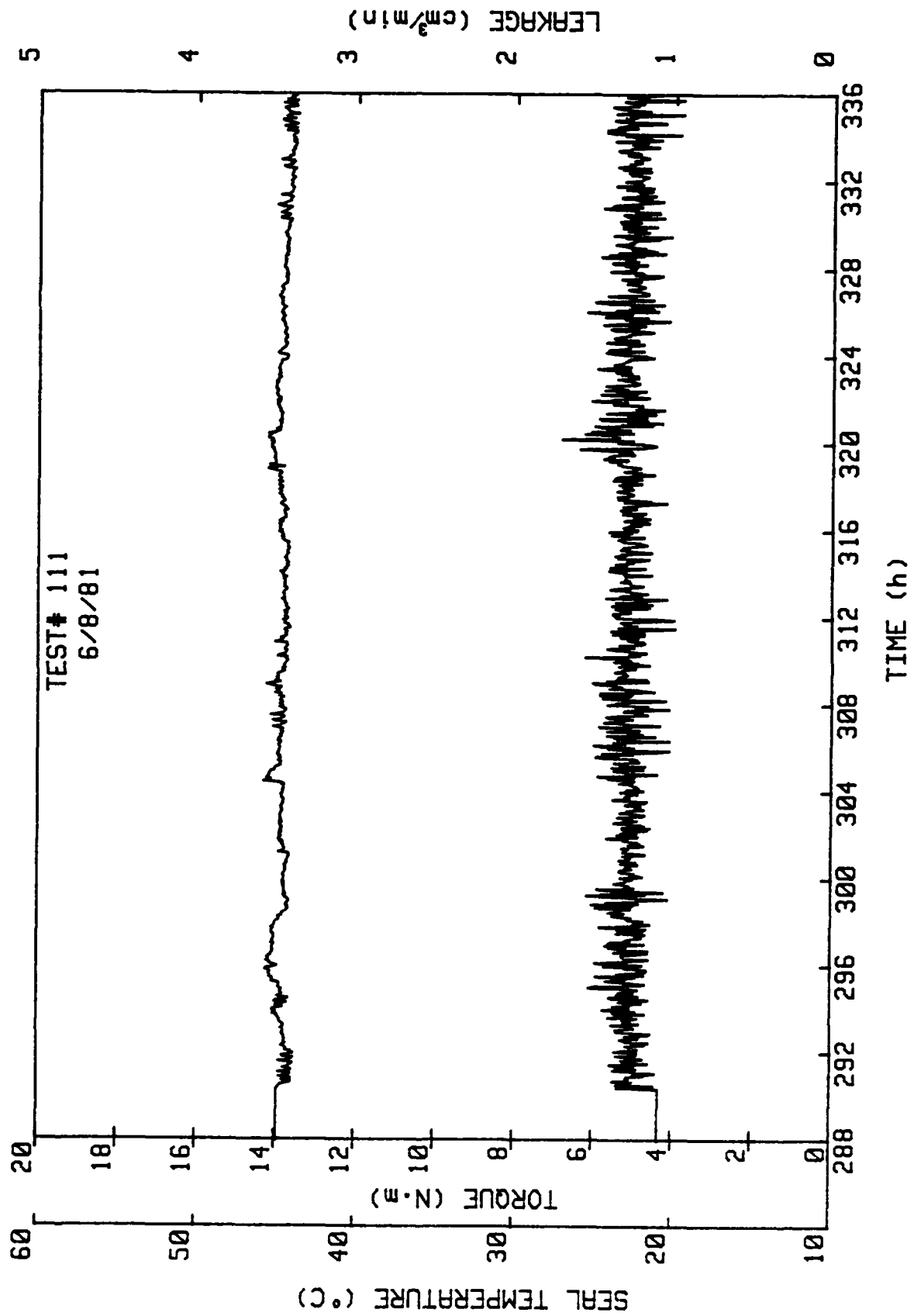


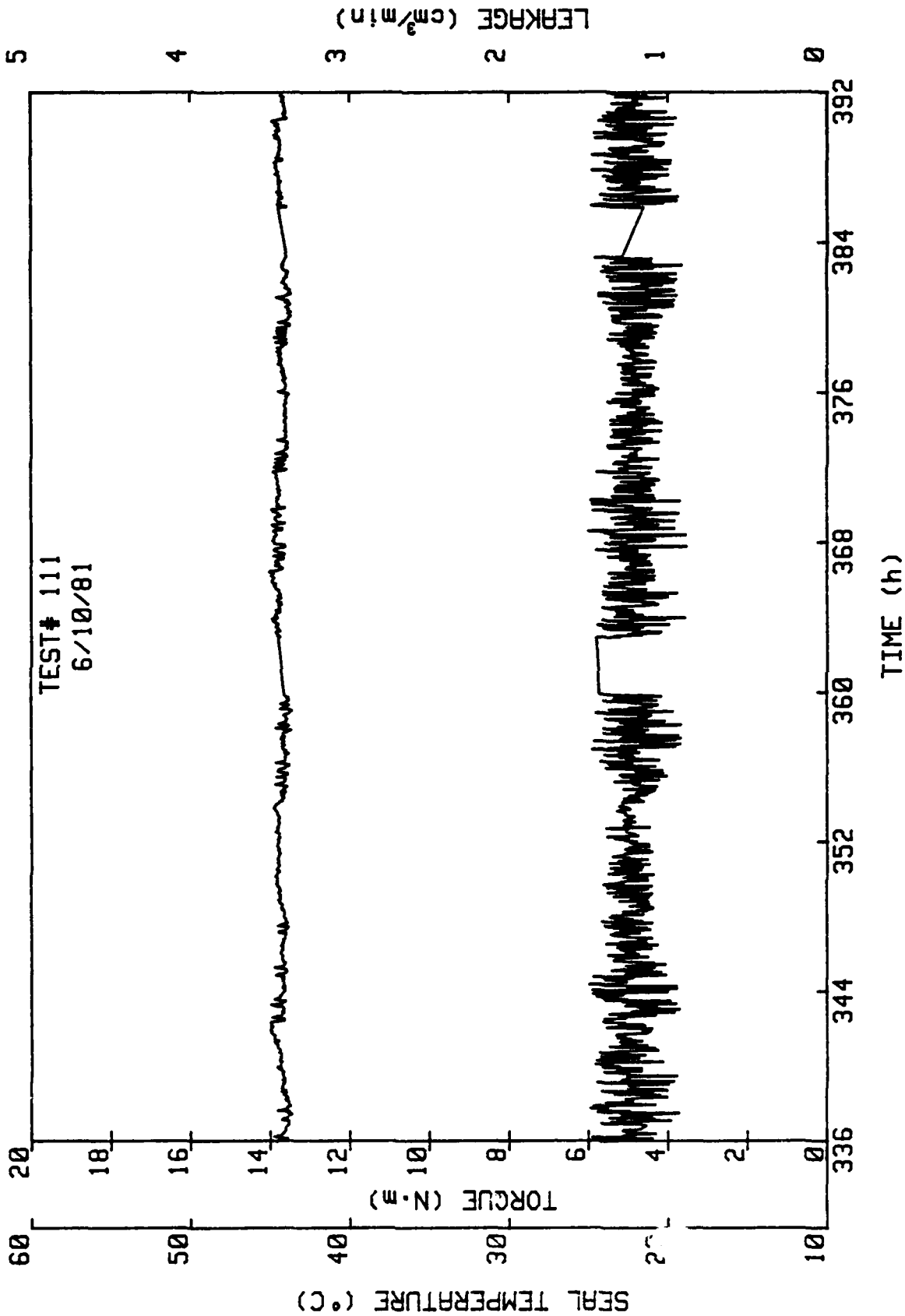


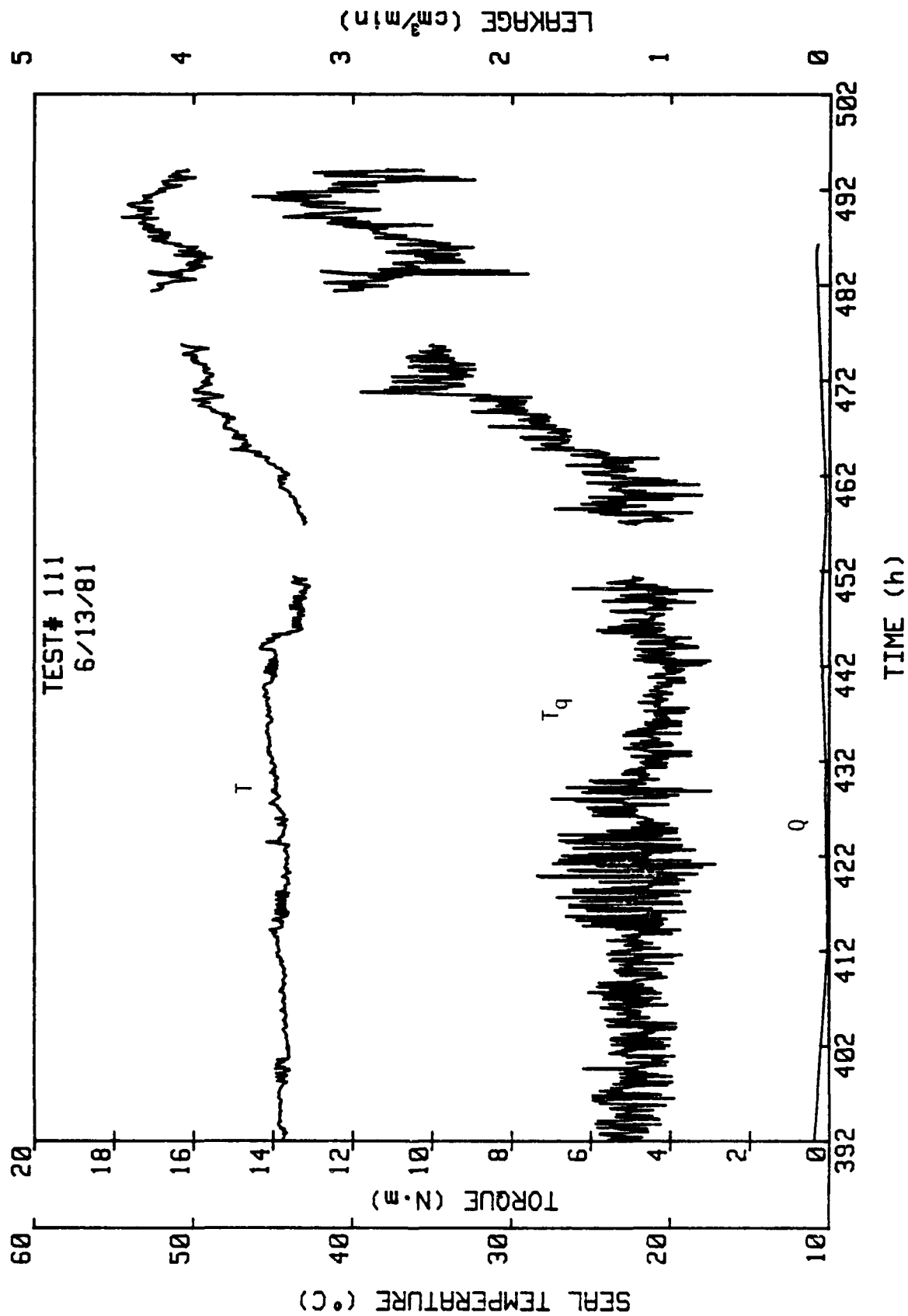












APPENDIX B  
DESIGN PROGRAM

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00010 C      THIS PROGRAM SOLVES FOR ALL DIMENSIONAL PARAMETERS FOR THE
00020 C      NEW SEAL DESIGN. IT INCORPORATES THE USE OF A FINITE
00030 C      DIFFERENCE METHOD TO SOLVE FOR THE TORSIONAL CONSTANT
00040 C      (JTHETA) AND ALSO UTILIZES A ROOT FINDING TECHNIQUE TO
00050 C      FIND THE ZERO OF (V/PHI) INITIAL AND (V/PHI) DESIRED.
00060 C      IT WAS FINISHED 12/10/80, A CHECK IS STILL NEEDED.
00070 C      THE PROGRAM WAS WRITTEN BY L.A. YOUNG.
00080 C
00090 C
00100      EXTERNAL F11
00110      COMMON NC,X4,X5,X7,Y5,X1,X2,X3,X6,X66,Y1,Y2,Y3,Y4,Y55
00120      COMMON Y6,Y7,O,PHI,AK,RI,RB,XX3,YY3,T1,T2,AJR
00130      COMMON A1,A2,A3,A4,A5,A6,A7,XBAR,YBAR,R20,RPI,PG
00140      COMMON PI,AMU,P,E,G,VOPHIG,VOPHI,ECC,A,A10,AJ0,V,EIX,AIX,V
00150      DOUBLE PRECISION X4,X5,X7,Y5,RL,RC,A11,B11,TOL1
00160      1000 WRITE(6,1)
00170      1   FORMAT(/,5X,'INPUT X1,X2,X66')
00180      READ(5,*) X1,X2,X66
00190      WRITE(6,2)
00200      2   FORMAT(/,5X,'INPUT Y1,Y2,Y3,Y55,Y6')
00210      READ(5,*) Y1,Y2,Y3,Y55,Y6
00220      WRITE(6,3)
00230      3   FORMAT(/,5X,'INPUT O,PHI')
00240      READ(5,*) O,PHI
00250      Y4=Y3
00260      AK=1.005
00270      RI=1.9
00280      RB=1.9
00290      XX3=.1875
00300      YY3=.125
00310      T1=.010
00320      T2=.003
00330      X3=XX3+2.*T1-1
00340      X5=.051
00350      Y7=.08
00360      PI=3.141592654
00370      AMU=.3
00380      P=.500.
00390      E=.9
00400      E=31.E+06
00410      G=E/2./(1.+AMU)
00420      PG=1000.0
00430      A11=1.200
00440      B11=1.500
00450      TOL1=1.00-6
00460      RL=ZEROIN(A11,B11,F11,TOL1)
00470      IF(A1.LT.0.0) GO TO 120
00480      IF(A2.LT.0.0) GO TO 120
00490      IF(A3.LT.0.0) GO TO 120
00500      IF(A4.LT.0.0) GO TO 120
00510      IF(A5.LT.0.0) GO TO 120
00520      IF(A6.LT.0.0) GO TO 120
00530      IF(A7.LT.0.0) GO TO 120
00540      GO TO 135
00550      120 WRITE(6,130) A1,A2,A3,A4,A5,A6,A7
00560      130  FORMAT(5X,'NEGATIVE AREAS ARE PRODUCED BY THOSE DIMENSIONS',/,5X
00570      *  ,',A1=',F8.5,2X,',A2=',F8.5,2X,',A3=',F8.5,2X,',A4=',F8.5,/,5X,',A5=',
00580      *  ,F8.5,2X,',A6=',F8.5,2X,',A7=',F8.5,///)
00590      GO TO 1000
00600      135  AIY=1./12.*X1**3*Y2+A1*(X1/2.-XBAR)**2
00610      AIY=AIY+1./12.*(X2-X1)**3*(Y1+Y2)+A2*(X1*(X2-X1)/2.-XBAR)**2
00620      AIY=AIY+1./12.*(X7-X1-X2)**3*(Y1+Y2-Y3)
00630      AIY=AIY+A3*(X2*(X7-X1-X2)/2.-XBAR)**2
00640      AIY=AIY+1./12.*X6**3*(Y5+Y6-Y4)+A4*(X1+X7+X5/2.-XBAR)**2
00650      AIY=AIY+1./12.*(X5-X4)**3*(Y5-Y4)

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00660      AIY=AIY+A5*(X1+X6+X7+(X5-X4)/2,-XBAR)*2
00670      AIY=AIY+1./12.*(X4-X66)**3+Y5+A6*(X2+X3+(X4-X66)/2,-XBAR)*2
00680      AIY=AIY+1./12.*X66**3*(Y5-Y55)+A7*(X2+X3+X4-X66/2,-XBAR)*2
00690      PL=ECC-YBAR
00700      AJR=AIY-AIX
00710      F1=2.*PI*(RL+X5+X6)*Y7
00720      F2=PI*(R00**2-R3**2)
00730      F3=2.*PI*(R00-X1)*Y1
00740      F5=2.*PI*(R00)*Y2
00750      F6=PI*((R00)**2-(R1+X*X3)**2)
00760      F7=2.*PI*(R1+X*X3)*Y*Y3
00770      F8=F2-F4-F6
00780      A1=F1*(YBAR=Y7/2.)
00790      R2=2./3.*(R00**3-R0**3)/(R00**2-R0**2)
00800      A12=F2*(R2=RC)
00810      A13=F3*(YBAR=Y1/2.)
00820      A15=F5*(Y1+Y2/2,-YBAR)
00830      R6=2./3.*(R00)**3-(R1+X*X3)**3)/((R00)**2-(R1+X*X3)**2)
00840      A16=F6*(R6=RC)
00850      A17=F7*(Y1+Y2+Y*Y3/2,-YBAR)
00860      R8=2./3.*X*X3+R1
00870      A18=F8*(R8=RC)
00880      T1=(A1+A12+A13+A14+A15+A16+A17+A18)/(2.*PI*RC)
00890      ROTATE=TH*RC**2/E/AIX*PH
00900      WRITE(6,150) XBAR,YBAR,AIX,AIY,AJR,AJ0,A,AM0,V,ECC,RC,EIX,ROTATE
00910      FORMAT(/,5X,'XBAR=',F7.4/,5X,'YBAR=',F7.4/,5X,'IX=',E12.5/,5X,
00920      * 'IY=',E12.5/,5X,'JR=',E12.5/,5X,'JTHETA=',E12.5/,5X,'A=',E12.5
00930      * '/',5X,'MTHETA=',E12.5/,5X,'V=',E12.5/,5X,'ECC=',E12.5/,5X,
00940      * 'RC=',F7.4/,5X,'EIX=',E12.5/,5X,'ROTATION=',E12.5)
00950      WRITE(6,160) V0PHIG,V0PHI,AIX,X1,X2,X3,X4,X5,X6,X7
00960      WRITE(6,170) Y1,Y2,Y3,Y4,Y5,Y6,Y7,R00,RPI,PL,RI,RL
00970      FORMAT(5X,'V/PHI=',E12.5,5X,'V/PHI=',E12.5/,5X,'IXY=',E12.5,
00980      * '/',5X,'X1=',F6.4,2X,'X2=',F6.4,2X,'X3=',F6.4,2X,'X4=',F6.4,2X,
00990      * 'X5=',F6.4,2X,'X6=',F6.4,2X,'X7=',F6.4)
01000      FORMAT(5X,'Y1=',F6.4,2X,'Y2=',F6.4,2X,'Y3=',F6.4,2X,'Y4=',F6.4
01010      * ',2X,'Y5=',F6.4,2X,'Y6=',F6.4,2X,'Y7=',F6.4/,5X,'R00=',F6.4,2X,
01020      * 'RPI=',F6.4,2X,'PL=',F6.4,2X,'RI=',F6.4,2X,'RL=',F6.4,///)
01030      GO TO 1000
01040      END
01050      DOUBLE PRECISION FUNCTION F11(RL)
01060
01070      COMMON RC,X4,X5,X7,Y5,X1,X2,X3,X6,X66,Y1,Y2,Y3,Y4,Y55
01080      COMMON Y6,Y7,D,PHI,AK,RI,R0,XX3,Y*Y3,T1,T2,AJR
01090      COMMON A1,A2,A3,A4,A5,A6,A7,XBAR,YBAR,R00,RPI,PG
01100      COMMON PI,AMU,PH,E,G,V0PHIG,V0PHI,ECC,A,A10,AJ0,V,EIX,AIX,V
01110      DOUBLE PRECISION X4,X5,X7,Y5,RC
01120      DIMENSION X(50),Y(50),P(50,50),AIN1(50,50),AIP1(50,50)
01130      DIMENSION AJP1(50,50),AJM1(50,50),BRMS(50,50)
01140      X4=RI-T1-RL
01150      X5=RB+T2-RL
01160      X7=-X1+X2+X3+X4-X5-X6
01170      Y5=Y1+Y2-Y3+Y4-Y6-Y7
01180      A1=X1+Y2
01190      A2=(X2-X1)*(Y1+Y2)
01200      A3=(X7+X1-X2)*(Y1+Y2-Y3)
01210      A4=X6*(Y5+Y6-Y4)
01220      A5=(X5-X4)*(Y5-Y4)
01230      A6=(X4-X66)*Y5
01240      A7=X66*(Y5-Y55)
01250      XBAR=A1*X1/2.+A2*(X1+(X2-X1)/2.)+A3*(X2+(X7+X1-X2)/2.)
01260      XBAR=XBAR+A4*(X1+X7+X6/2.)+A5*(X1+X6+X1/(X5-X4)/2.)
01270      XBAR=XBAR+A6*(X2+X3+(X4-X66)/2.)+A7*(X2+X3+X4-X66/2.)
01280      XBAR=XBAR/(A1+A2+A3+A4+A5+A6+A7)
01290
01300      YBAR=A1*(Y1+Y2/2.)+A2*(Y1+Y2)/2.+A3*(Y1+Y2-Y3)/2.
01310      YBAR=YBAR+A4*(Y7+(Y6+Y5-Y4)/2.)+A5*(Y6+Y7+(Y5-Y4)/2.)
01320

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01330 YBAR=YBAR+A6*(Y5+Y7+Y5/2.)*A7*(Y55+Y6+Y7+(Y5-Y55)/2.)
01340 YBAR=YBAR/(A1+A2+A3+A4+A5+A6+A7)
01350 C
01360 AIX=1./12.*X1+Y2**3+A1*(Y1+Y2/2.-YBAR)**2
01370 AIX=AIX+1./12.*(X2-X1)*(Y1+Y2)**3+A2*((Y1+Y2)/2.-YBAR)**2
01380 AIX=AIX+1./12.*(X7+X1-X2)*(Y1+Y2+Y3)**3+A3*((Y1+Y2+Y3)/2.-YBAR)**
01390 *2
01400 AIX=AIX+1./12.*X6*(Y5+Y6+Y4)**3+A4*(Y7+(Y5+Y6+Y4)/2.-YBAR)**2
01410 AIX=AIX+1./12.*(X5-X4)*(Y5+Y4)**3+A5*(Y6+Y7+(Y5+Y4)/2.-YBAR)**2
01420 AIX=AIX+1./12.*(X4-X66)*Y5**3+A6*(Y6+Y7+Y5/2.-YBAR)**2
01430 AIX=AIX+1./12.*X66*(Y5-Y55)**3+A7*(Y55+Y6+Y7+(Y5-Y55)/2.-YBAR)**2
01440 C
01450 R20=X2+X3+X4+RL
01460 - RPI=RL+X5+X6+X7/2. ←
01470 RC=R20-XBAR
01480 C
01490 C
01500 EIX=E*AIX
01510 ERR=0.000001
01520 AA1=X66
01530 AA2=X4
01540 AA3=X5
01550 AA4=X5+X6
01560 AA5=X5+X6+X7
01570 AA6=X1+X5+X6+X7
01580 B1=Y7
01590 B2=Y1
01600 B3=Y6+Y7
01610 B4=Y1+Y2+Y3
01620 B5=Y55+Y6+Y7
01630 B6=Y1+Y2
01640 I1=4
01650 I2=13
01660 I3=17
01670 I4=19
01680 I5=20
01690 I6=22
01700 J1=17
01710 J2=23
01720 J3=24
01730 J4=26
01740 J5=30
01750 J6=37
01760 XX=0.0
01770 X(1)=0.0
01780 OX=B1/(I1-1)
01790 DO 10 I=2,I6
01800 IF(I,GT,I1) OX=(B2-B1)/(I2-I1)
01810 IF(I,GT,I2) OX=(B3-B2)/(I3-I2)
01820 IF(I,GT,I3) OX=(B4-B3)/(I4-I3)
01830 IF(I,GT,I4) OX=(B5-B4)/(I5-I4)
01840 IF(I,GT,I5) OX=(B6-B5)/(I6-I5)
01850 XX=XX+OX
01860 X(I)=XX
01870 10 CONTINUE
01880 YY=0.0
01890 Y(1)=0.0
01900 OY=AA1/(J1-1)
01910 DO 20 J=2,J6
01920 IF(J,GT,J1) OY=(AA2-AA1)/(J2-J1)
01930 IF(J,GT,J2) OY=(AA3-AA2)/(J3-J2)
01940 IF(J,GT,J3) OY=(AA4-AA3)/(J4-J3)
01950 IF(J,GT,J4) OY=(AA5-AA4)/(J5-J4)
01960 IF(J,GT,J5) OY=(AA6-AA5)/(J6-J5)
01970 YY=YY+OY
01980 Y(J)=YY

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01990      20  CONTINUE
02000      IYAX=I6-1
02010      JYAX=J6-1
02020      D0 30 I=2,IYAX
02030      D0 30 J=2,JYAX
02040      AIJ=2./((X(I-1)-X(I))/(X(I+1)-X(I))+2./((Y(J-1)-Y(J))/(Y(J+1)-Y(J))
02050      *)
02060      AIM1(I,J)=2./((X(I-1)-X(I))/(X(I-1)-X(I+1)))/AIJ
02070      AIP1(I,J)=2./((X(I+1)-X(I))/(X(I-1)-X(I+1)))/AIJ
02080      AJM1(I,J)=2./((Y(J-1)-Y(J))/(Y(J-1)-Y(J+1)))/AIJ
02090      AJP1(I,J)=2./((Y(J+1)-Y(J))/(Y(J-1)-Y(J+1)))/AIJ
02100      BRMS(I,J)=2./AIJ
02110      30  CONTINUE
02120      D0 40 I=1,I6
02130      D0 40 J=1,J6
02140      40  P(I,J)=0.000000000000001
02150      QYEG=1.5
02160      QY0=1.-QYEG
02170      D0 60 IT=1,200
02180      AYAX=0.0
02190      D0 50 J=2,JYAX
02200      IYAX=I6-1
02210      IMIN=I5+1
02220      IF(J,GT,J1) IMIN=I3+1
02230      IF(J,GT,J2) IYAX=I4-1
02240      IF(J,GT,J3) IMIN=I1+1
02250      IF(J,GT,J4) IMIN=2
02260      IF(J,GT,J5) IMIN=I2+1
02270      IF(J,GT,J5) IYAX=I6-1
02280      D0 50 I=IMIN,IYAX
02290      P0L0=P(I,J)
02300      PNEW=AIM1(I,J)*P(I-1,J)+AIP1(I,J)*P(I+1,J)+AJM1(I,J)*P(I,J-1)
02310      PNEW=PNEW+AJP1(I,J)*P(I,J+1)+BRMS(I,J)
02320      P(I,J)=PNEW*QYEG+QY0*P0L0
02330      VAL=ABS((P(I,J)-P0L0)/P0L0)
02340      IF(VAL,GT,AYAX) AYAX=VAL
02350      50  CONTINUE
02360      IF(VAL,LT,ERR) G0 T0 70
02370      60  CONTINUE
02380      WRITE(6,65)
02390      65  FORMAT(2X,'EXCEEDS ITERATIONS')
02400      70  JYAX=J6-1
02410      IYAX=I6-1
02420      AJ=0.0
02430      D0 80 I=1,IYAX
02440      D0 90 J=1,JYAX
02450      Z=P(I,J)+P(I+1,J)+P(I+1,J+1)+P(I,J+1)
02460      80  AJ=AJ+.25*Z*((X(I+1)-X(I))*((Y(J+1)-Y(J))
02470      J=J6+1
02480      AJ=2.*AJ
02490      AJ0=AJ
02500      A=E*AIJ/G/AJ0
02510      C
02520      AY0=(PHI+E*AIJ)/RC**2*(Y**2-1.)**2/(4*Y**2+1.)
02530      ECC=AM0/D*RC/20I/2G/SIN(Y*0/2./RPI)
02540      V=RC**3*AM0/E/AIX*(1.+A)/(Y**2-1.)**2
02550      V0PHIG=V/PHI
02560      V0PHI=RC*AK*RI
02570      F11=V0PHIG-V0PHI
02580      RETURN
02590      END
02600      DOUBLE PRECISION FUNCTION ZERBIN(AX,BX,F,T0L)
02610      DOUBLE PRECISION AX,BX,F,T0L
02620      DOUBLE PRECISION A,B,C,D,E,EPS,FA,F0,FC,F0L1,X4,P,4,4,9
02630      DOUBLE PRECISION JAS5
02640      C

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02650 C: COMPUTE EPS, THE RELATIVE MACHINE PRECISION
02660 C:
02670 EPS = 1.000
02680 10 EPS = EPS/2.000
02690 TOL1 = 1.000 + EPS
02700 IF (TOL1 .GT. 1.000) GO TO 10
02710 C:
02720 C: INITIALIZATION
02730 C:
02740 A = AX
02750 B = BX
02760 FA = F(A)
02770 FB = F(B)
02780 C:
02790 C: BEGIN STEP
02800 C:
02810 20 C = A
02820 FC = FA
02830 D = B - A
02840 E = D
02850 30 IF (DABS(FC) .GE. DABS(FB)) GO TO 40
02860 A = B
02870 B = C
02880 C = A
02890 FA = FB
02900 FB = FC
02910 FC = FA
02920 C:
02930 C: CONVERGENCE TEST
02940 C:
02950 40 TOL1 = 2.000*EPS*DABS(B) + 0.500*TOL
02960 XM = .5*(C + B)
02970 IF (DABS(XM) .LE. TOL1) GO TO 90
02980 IF (FB .EQ. 0.000) GO TO 90
02990 C:
03000 C: IS BISECTION NECESSARY
03010 C:
03020 IF (DABS(E) .LT. TOL1) GO TO 70
03030 IF (DABS(FA) .LE. DABS(FB)) GO TO 70
03040 C:
03050 C: IS QUADRATIC INTERPOLATION POSSIBLE
03060 C:
03070 IF (A .NE. C) GO TO 50
03080 C:
03090 C: LINEAR INTERPOLATION
03100 C:
03110 S = FB/FA
03120 P = 2.000*XM*S
03130 Q = 1.000 - S
03140 GO TO 60
03150 C:
03160 C: INVERSE QUADRATIC INTERPOLATION
03170 C:
03180 50 Q = FA/FC
03190 R = FB/FC
03200 S = FB/FA
03210 P = S*(2.000*XM*Q*(Q + R) + (B - A)*(Q - 1.000))
03220 Q = (Q + 1.000)*(R + 1.000)*(S + 1.000)
03230 C:
03240 C: ADJUST SIGNS
03250 C:
03260 60 IF (P .GT. 0.000) Q = -Q
03270 P = DABS(P)
03280 C:
03290 C: IS INTERPOLATION ACCEPTABLE
03300 C:

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03310 IF ((2.000*P) .GE. (3.000*X+Q - DABS(T0L1*Q))) G0 T0 70
03320 IF (P .GE. DABS(0.500*E*2)) G0 T0 70
03330 E = 0
03340 J = P/Q
03350 G0 T0 80
03360 C
03370 C BISECTION
03380 C
03390 70 D = XM
03400 E = 0
03410 C
03420 C COMPLETE STEP
03430 C
03440 80 A = B
03450 FA = FB
03460 IF (DABS(0) .GT. T0L1) B = B + D
03470 IF (DABS(0) .LE. T0L1) B = B + USIGN(T0L1, XM)
03480 FB = F(B)
03490 IF ((FB*(FC/DABS(FC))) .GT. 0.000) G0 T0 20
03500 G0 T0 30
03510 C
03520 C DONE
03530 C
03540 90 ZEROIN = B
03550 RETURN
03560 END

```

```

10 C PROGRAM NAVSEAL1
20 C DESIGN MODIFICATION PROGRAM TO LOWER THE TORSIONAL STIFFNESS OF
30 C THE EXISTING SEAL RING.
40 C
50     DIMENSION X(50),Y(50),P(50,50),AIM1(50,50),AIP1(50,50)
60     DIMENSION AJP1(50,50),AJM1(50,50),BRHS(50,50)
70     REAL K
80     5 WRITE(6,10)
90     10 FORMAT(/,5X,'INPUT X1,Y1')
100    READ(5,*) X1,Y1
110    WRITE(6,20)
120    20 FORMAT(/,5X,'INPUT X2,X3,Y4')
130    READ(5,*)X2,X3,Y4
140    WRITE(6,30)
150    30 FORMAT(/,5X,'INPUT D,Y6')
160    READ(5,*)D,Y6
170    K=1.015
180    RI=1.9
190    RL=1.9
200    RL=1.389
210    XX3=.1875
220    YY3=.125
230    T1=.010
240    T2=.003
250    X7=.36
260    X8=.154
270    X9=.051
280    X10=.1435-X1
290    X4=X2-X3
300    X5=XX3+2.*T1
310    31 X6=RI-T1-RL
320    Y2=.38-Y1
330    Y3=.22-Y4
340    Y5=.1
350    Y7=.09-(.1-Y6)
360    Y8=.08
370    Y9=.35
380    Y10=.08
390    PI=3.141592654
400    AMU=.278
410    PW=500.
420    PL=.5602
430    N=9
440    E=31.E+06
450    G=E/2./(.1+AMU)
460    PG=750.0
470    100 A1=X3*Y3
480    A2=(Y3+Y4)*X4
490    A3=X1*(Y2+Y3+Y4-Y5)
500    A4=X10*(Y1+Y2+Y3+Y4-Y5)
510    A5=X9*(Y1+Y2+Y3+Y4-Y5-Y10)
520    A6=(X7+X8-X6)*(Y7+Y8-Y6)
530    A7=(X6-X7)*(Y7+Y8)
540    A8=X7*Y7
550    IF(A1.LT.0.0) GO TO 36
560    IF(A2.LT.0.0) GO TO 36

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570      IF(A3.LT.0.0) GO TO 36
580      IF(A4.LT.0.0) GO TO 36
590      IF(A5.LT.0.0) GO TO 36
600      IF(A6.LT.0.0) GO TO 36
610      IF(A7.LT.0.0) GO TO 36
620      IF(A8.LT.0.0) GO TO 36
630      FXA1=X3/2.
640      FXA2=X3+X4/2.
650      FXA3=X2+X1/2.
660      FXA4=X1+X2+X10/2.
670      FXA5=X1+X2+X10+X9/2.
680      FXA6=X1+X2+X10+X9+(X3+X4+X5-X2-X1-X10-X9)/2.
690      FXA7=X3+X4+X5+(X6-X7)/2.
700      FXA8=X3+X4+X5+X6-X7/2.
710 C
720      FYA1=Y1+Y2+Y3/2.
730      FYA2=Y1+Y2+(Y3+Y4)/2.
740      FYA3=Y1+(Y2+Y3+Y4-Y5)/2.
750      FYA4=(Y1+Y2+Y3+Y4-Y5)/2.
760      FYA5=Y10+(Y1+Y2+Y3+Y4-Y5-Y10)/2.
770      FYA6=Y10+Y9+(Y8+Y7-Y6)/2.
780      FYA7=Y10+Y9+(Y8+Y7)/2.
790      FYA8=Y10+Y9+Y8+Y7/2.
800 C
810      XBAR=A1*FXA1+A2*FXA2+A3*FXA3+A4*FXA4
820      XBAR=XBAR+A5*FXA5+A6*FXA6+A7*FXA7+A8*FXA8
830      XBAR=XBAR/(A1+A2+A3+A4+A5+A6+A7+A8)
840 C
850      YBAR=A1*FYA1+A2*FYA2+A3*FYA3+A4*FYA4
860      YBAR=YBAR+A5*FYA5+A6*FYA6+A7*FYA7+A8*FYA8
870      YBAR=YBAR/(A1+A2+A3+A4+A5+A6+A7+A8)
880 C
890      AIX=1./12.*X3**3+A1*(FYA1-YBAR)**2
900      AIX=AIX+1./12.*X4*(Y3+Y4)**3+A2*(FYA2-YBAR)**2
910      AIX=AIX+1./12.*X1*(Y2+Y3+Y4-Y5)**3+A3*(FYA3-YBAR)**2
920      AIX=AIX+1./12.*X10*(Y1+Y2+Y3+Y4-Y5)**3+A4*(FYA4-YBAR)**2
930      AIX=AIX+1./12.*X9*(Y1+Y2+Y3+Y4-Y5-Y10)**3+A5*(FYA5-YBAR)**2
940      AIX=AIX+1./12.*(X8-(X6-X7))*(Y7+Y8-Y6)**3+A6*(FYA6-YBAR)**2
950      AIX=AIX+1./12.*(X8)*(Y7+Y8)**3+A7*(FYA7-YBAR)**2
960      AIX=AIX+1./12.*X7*(Y7)**3+A8*(FYA8-YBAR)**2
970 C
980      AIY=1./12.*X3**3*Y3+A1*(FXA1-XBAR)**2
990      AIY=AIY+1./12.*X4**3*(Y3+Y4)+A2*(FXA2-XBAR)**2
1000     AIY=AIY+1./12.*X1**3*(Y2+Y3+Y4-Y5)+A3*(FXA3-XBAR)**2
1010     AIY=AIY+1./12.*X10**3*(Y1+Y2+Y3+Y4-Y5)+A4*(FXA4-XBAR)**2
1020     AIY=AIY+1./12.*X9**3*(Y1+Y2+Y3+Y4-Y5-Y10)+A5*(FXA5-XBAR)**2
1030     AIY=AIY+1./12.*(X8-(X6-X7))**3*(Y7+Y8-Y6)+A6*(FXA6-XBAR)**2
1040     AIY=AIY+1./12.*X8**3*(Y7+Y8)+A7*(FXA7-XBAR)**2
1050     AIY=AIY+1./12.*X7**3*Y7+A8*(FXA8-XBAR)**2
1060 C
1070     R00=RL+X6+X5+X4+X3
1080     RC=R00-XBAR
1090     RPI=2.484
1100 C
1110 C
1120     AJR=AIX+AIY
1130     EIX=E*AIX
1140     AJO=(.025*(A1+A2+A3+A4+A5+A6+A7+A8)**4)/AJR
1150 C TORSIONAL CONSTANT CALCULATION USING FINITE DIFFERENCE
1160     ERR=0.0000001
1170     AA1=X7
1180     AA2=X6
1190     AA3=X7+X8
1200     AA4=X7+X8+X9
1210     AA5=X7+X8+X9+X10
1220     AA6=X5+X6

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1230      AA7=X4+X5+X6
1240      AA8=X3+X4+X5+X6
1250      BB1=Y10
1260      BB2=Y1
1270      BB3=Y1+Y2
1280      BB4=Y9+Y10
1290      BB5=Y1+Y2+Y3+Y4-Y5
1300      BB6=Y8+Y9+Y10
1310      BB7=Y1+Y2+Y3
1320      BB8=Y7+Y8+Y9+Y10
1330      BB9=Y1+Y2+Y3+Y4
1340      I1=4
1350      I2=5
1360      I3=13
1370      I4=17
1380      I5=19
1390      I6=20
1400      I7=21
1410      I8=22
1420      I9=23
1430      J1=17
1440      J2=23
1450      J3=24
1460      J4=26
1470      J5=29
1480      J6=30
1490      J7=36
1500      J8=37
1505 C MESH GENERATION ROUTINE
1510      XX=0.0
1520      X(1)=0.0
1530      DX=BB1/(I1-1)
1540      DO 60 I=2, I9
1550          IF(I.GT.I1) DX=(BB2-BB1)/(I2-I1)
1560          IF(I.GT.I2) DX=(BB3-BB2)/(I3-I2)
1570          IF(I.GT.I3) DX=(BB4-BB3)/(I4-I3)
1580          IF(I.GT.I4) DX=(BB5-BB4)/(I5-I4)
1590          IF(I.GT.I5) DX=(BB6-BB5)/(I6-I5)
1600          IF(I.GT.I6) DX=(BB7-BB6)/(I7-I6)
1610          IF(I.GT.I7) DX=(BB8-BB7)/(I8-I7)
1620          IF(I.GT.I8) DX=(BB9-BB8)/(I9-I8)
1630          XX=XX+DX
1640          X(I)=XX
1650      60 CONTINUE
1660      YY=0.0
1670      Y(1)=0.0
1680      DY=AA1/(J1-1)
1690      DO 70 J=2, J8
1700          IF(J.GT.J1) DY=(AA2-AA1)/(J2-J1)
1710          IF(J.GT.J2) DY=(AA3-AA2)/(J3-J2)
1720          IF(J.GT.J3) DY=(AA4-AA3)/(J4-J3)
1730          IF(J.GT.J4) DY=(AA5-AA4)/(J5-J4)
1740          IF(J.GT.J5) DY=(AA6-AA5)/(J6-J5)
1750          IF(J.GT.J6) DY=(AA7-AA6)/(J7-J6)
1760          IF(J.GT.J7) DY=(AA8-AA7)/(J8-J7)
1770          YY=YY+DY
1780          Y(J)=YY
1790      70 CONTINUE
1800      IMAX=I9-1
1810      JMAX=J8-1
1820      DO 80 I=2, IMAX
1830          DO 80 J=2, JMAX
1840              RIJ=-2./(X(I-1)-X(I))/(X(I+1)-X(I))-2./(Y(J-1)-Y(J))/
1850              * (Y(J+1)-Y(J))
1860              RIM1(I,J)=2./(X(I-1)-X(I))/(X(I-1)-X(I-1))/RIJ
1870              RIP1(I,J)=-2./(X(I+1)-X(I))/(X(I-1)-X(I-1))/RIJ

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1880      AJM1(I, J)=2. / (Y(J-1)-Y(J)) / (Y(J-1)-Y(J+1)) / AIJ
1890      AJP1(I, J)=-2. / (Y(J+1)-Y(J)) / (Y(J-1)-Y(J+1)) / AIJ
1900      BRHS(I, J)=2. / AIJ
1910 80 CONTINUE
1920      DO 90 I=1, I9
1930      DO 90 J=1, J8
1940          P(I, J)=0. 00000000000001
1950 90 CONTINUE
1960      OMEG=1. 5
1970      OMO=1. -OMEG
1980      DO 95 IT=1, 800
1990          AMAX=0. 0
2000      DO 94 J=2, JMAX
2010          IMAX=I8-1
2020          IMIN=I6+1
2030          IF(J. GT. J1) IMIN=I4+1
2040          IF(J. GT. J2) IMAX=I5-1
2050          IF(J. GT. J3) IMIN=I1+1
2060          IF(J. GT. J4) IMIN=2
2070          IF(J. GT. J5) IMIN=I2+1
2080          IF(J. GT. J6) IMIN=I3+1
2090          IF(J. GT. J6) IMAX=I9-1
2100          IF(J. GT. J7) IMAX=I7-1
2110      DO 94 I=IMIN, IMAX
2120          POLD=P(I, J)
2130          PNEW=AIM1(I, J)*P(I-1, J)+AIP1(I, J)*P(I+1, J)
2140          PNEW=PNEW+AJM1(I, J)*P(I, J-1)+AJP1(I, J)*P(I, J+1)+BRHS(I, J)
2150          P(I, J)=PNEW*OMEG+OMO*POLD
2160          VAL=ABS((P(I, J)-POLD)/POLD)
2170          IF(VAL. GT. AMAX) AMAX=VAL
2180 94 CONTINUE
2190      IF(VAL. LT. ERR) GO TO 110
2200 95 CONTINUE
2210      WRITE(6, 96)
2220 96 FORMAT(2X, 'EXCEEDS ITERATIONS')
2230 110 JMAX=J8-1
2240      IMAX=I8-1
2250      AJ=0. 0
2255 C INTEGRATION ROUTINE
2260      DO 115 I=1, IMAX
2270      DO 115 J=1, JMAX
2280          Z=P(I, J)+P(I+1, J)+P(I+1, J+1)+P(I, J+1)
2290          AJ=AJ+. 25*Z*(X(I+1)-X(I))*(Y(J+1)-Y(J))
2300 115 CONTINUE
2310      J=J8+1
2320      AJ=2. *AJ
2330      AJO=AJ
2340 116 CONTINUE
2350      A=E*AIJ/G/AJO
2360      ECC=PL+YBAR
2365 C WARPING FACTOR CALCULATION (AJMEGA).
2370      XTOT=X3+X4+X5+X6
2380      T1AV=Y7*(X7/XTOT)+(Y7+Y8)*((X6-X7)/XTOT)+(Y8+Y7-Y6)*(X5/XTOT)
2390      * +(Y3+Y4)*(X4/XTOT)+Y3*(X2/XTOT)
2400      YTOT=Y9+Y10
2410      T2AV=X10*(Y10/YTOT)+(X9+X10)*((Y1-Y10)/YTOT)+(X1+X9+X10)
2420      * *((Y2+Y3+Y4)-(Y7+Y8))/YTOT
2430      H=Y10+Y9+T1AV/2.
2440      BDIM=XTOT
2450      AJMEGA=T1AV**3*BDIM**3/144+T2AV**3*H**3/36
2460      B=AIJ*RC**2/AJMEGA
2470      FAC1=N**2/A+N**4/B+N**2
2480      FAC2=N**2+1. /A+N**2/B
2490      FAC3=1. +1. /A+N**2/B
2500      FAC4=N**2/A+N**4/B+1.
2510      AMO=ECC*0. *RPI*PG*SIN(N*0/(2. *RPI))/RC

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2520      PHI=AMO*RC**2/E/RIX*(1./((N**2/A+N**4/B+1.)-(1.+1./A+N**2/B)
2530      */(N**2+1./A+N**2/B)*(N**2/A+N**4/B+N**2)))
2540      V=AMO*RC**2/E/RIX*(1./((FAC1-FAC2/FAC3*FAC4)))
2550      RSEAL=RC-V/PHI
2555 C PRESSURE MOMENT CALCULATION
2560      F1=2.*PI*(RL+X7+X8+X9)*Y10
2570      F2=PI*(R00**2-RB**2)
2580      F3=2.*PI*(RL+X7+X8+X9+X10)*Y1
2590      F4=2.*PI*(RL+X7+X8+X9+X10+X1)*Y2
2600      F5=2.*PI*(R00)*Y3
2610      F6=PI*(R00**2-(R00-X3)**2)
2620      F7=2.*PI*(R00-X3)*Y4
2630      F8=PI*((R00-X3)**2-(R00-(X4+X3+T1))**2)
2640      F9=2.*PI*(RI+XX3)*YY3
2650      F10=F2-F6-F8
2660      AM1=-F1*(YBAR-Y10/2.)
2670      R2=2./3.*(R00**3-RB**3)/(R00**2-RB**2)
2680      AM2=-F2*(R2-RC)
2690      AM3=F3*(YBAR-Y1/2.)
2700      AM4=F4*(YBAR-(Y1+Y2/2.))
2710      AM5=-F5*(Y1+Y2+Y3/2.-YBAR)
2720      R6=2./3.*(R00**3-(R00-X3)**3)/(R00**2-(R00-X3)**2)
2730      AM6=F6*(R6-RC)
2740      AM7=-F7*(Y1+Y2+Y3+Y4/2.-YBAR)
2750      R8=2./3.*((R00-X3)**3-(R00-X3-X4-T1)**3)/((R00-X3)**2-(R00-X3-
2760      *X4-T1)**2)
2770      AM8=F8*(R8-RC)
2780      AM9=-F9*(Y2+Y1+Y3+Y4+YY3/2.-YBAR)
2790      R10=2./3.*XX3+RI
2800      AM10=F10*(R10-RC)
2810      TM=(AM1+AM2+AM3+AM4+AM5+AM6+AM7+AM8+AM9+AM10)/(2.*PI*RC)
2820      WRITE(6,*) B
2830      ROTATE=TM*RC**2/E/RIX*PW
2840      GO TO 39
2850 36 WRITE(6,37)A1,A2,A3,A4,A5,A6
2860 37 FORMAT(5X,'NEGATIVE AREAS ARE PRODUCED BY THOSE DIMENSIONS',/,5X
2870      *,'A1=',F6.4,2X,'A2=',F6.4,2X,'A3=',F6.4,2X,'A4=',F6.4,2X,'A5=',
2880      *F6.4,2X,'A6=',F6.4,///)
2890      GO TO 5
2900 39 WRITE(6,40) XBAR,YBAR,RIX,RIY,AJR,AJO,A,AMO,V,ECC,RC,EIX,ROTATE
2910 40 FORMAT(/,5X,'XBAR=',F7.4,/,5X,'YBAR=',F7.4,/,5X,'IX=',E12.5,/,5X,
2920      *,'IY=',E12.5,/,5X,'JR=',E12.5,/,5X,'JTHETA=',E12.5,/,5X,'A=',E12.5
2930      */,5X,'MTHETA=',E12.5,/,5X,'V=',E12.5,/,5X,'ECC=',E12.5,/,5X,
2940      *,'RC=',F7.4,/,5X,'EIX=',E12.5,/,5X,'ROTATION=',E12.5)
2950      WRITE(6,45) RSEAL,RIX,Y1,X2,X3,X4,/,5,X6,X7,X8,X9,X10,PHI
2960      WRITE(6,50) Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8,Y9,Y10,R00,RPI,PL,RI,RL
2970 45 FORMAT(5X,'RSEAL=',E12.5,/,5X,'IXY=',E12.5,
2980      */,5X,'X1=',F6.4,2X,'X2=',F6.4,2X,'X3=',F6.4,2X,'X4=',F6.4,2X,
2990      *,'X5=',F6.4,2X,'X6=',F6.4,2X,'X7=',F6.4,/,5X,'X8=',F6.4,2X,'X9=',
3000      *F6.4,2X,'X10=',F6.4,/,5X,'PHI=',E12.5)
3010 50 FORMAT(5X,'Y1=',F6.4,2X,'Y2=',F6.4,2X,'Y3=',F6.4,2X,'Y4=',F6.4
3020      *,'Y5=',F6.4,2X,'Y6=',F6.4,2X,'Y7=',F6.4,/,5X,'Y8=',F6.4
3030      *,'Y9=',F6.4,2X,'Y10=',F6.4,/,5X,'R00=',F6.4,2X,
3040      *,'RPI=',F6.4,2X,'PL=',F6.4,2X,'RI=',F6.4,2X,'RL=',F6.4,///)
3050      GO TO 5
3060      END

```



APPENDIX C  
GAS SEAL PROGRAM

\*\*\*\* Program Listing \*\*\*\*

The fluid used is the nitrogen gas.

Alternating Direction Implicit technique is used to analyze a moving wave gas seal.

A set of simultaneous equations at each time step are solved by a special Gaussian Elimination Subroutine, in which a tridiagonal coefficient matrix is rearranged in a form of three columns matrix. The advantage is to save a lot of computer space, i.e.  $n \times n$  storages decreases to  $n \times 3$  storages.

Implicit real\*8 is used to convert all real single precision numbers to double precision numbers.

The seal is divided into three identical parts (3 waves), each part is subdivided into  $19 \times 20$  nodes, where 19 nodes are in circumferential directions and 20 nodes are in radial directions.

The values used in this program have the following units:

C	B	: Balanced ratio	
C	c	: One-half maximum roughness height	:in
C	DT	: Time step	:sec
C	ETA	: Viscosity of nitrogen	:lbf-sec/in/in
C	EM	: Young's Modulus	:psi
C	GM	: Shear Modulus	:psi
C	G	: Gravity	:ft/sec/sec
C	HA	: Ring centroid amplitude	:in
C	PC	: Cavity pressure	:psi
C	PM	: Pressure at asperity contact (equals compressive strength)	:psi
C	PI	: Seal inside pressure	:psi
C	PO	: Seal outside pressure	:psi
C	PSP	: Spring pressure on face	:psi
C	PSBAR	: Shear strength of pressure	:psi
C	PHI	: Tilt	:in/in
C	RB	: Seal balance radius	:in
C	RC	: Seal centroid radius	:in
C	RI	: Inside radius	:in
C	RO	: Outside radius	:in
C	RF	: Friction radius	:in
C	RHO	: Density	:in
C	RPM	: Shaft angular speed	:rpm
C	TEMP	: temperature	:°R
C	UCON	: Universal constant	:ft-lbf/lbm-R°

C  
C  
C  
C  
C

The main program

```

IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION NO(18)
COMMON /BLOCKA/C,DA,DEM,DR,ETA,OMEG,PI,PO,RO,WE,IMAX,JMAX
COMMON /BLOCKB/AN,DT,HA,PHI,RC,WW,NIT
COMMON /BLOCKC/H(18,20),P(18,20),Q(18,20),RR(18,20)
COMMON /BLOCKD/AMAXER,RI,NN,IM1,JM1
COMMON /BLOCKE/PFAC,PHLS,PM,TOTLD,WMECH
COMMON /BLOCKF/FCONV,WSTAR,ROOC,NWAVE
IMAX=19
JMAX=20
JM1=JMAX-1
IM1=IMAX-1
NWAVE=3
NN=NWAVE
RC=1.9361D0
HA=5.4D-6
PIE=3.141592654D0
EM=3.D+6
GM=EM/2.4D0
AN=2.D0*PIE/NN
DA=AN/IM1
AMAXER=6.0D0
B=0.75D0
C=20.D-6
DT=0.5D-6
PSP=28.5D0
PC=0.0D0
G=32.174D0
RI=1.900D0
RO=2.0875D0
WLIMIT=3.0D0
DR=(RO-RI)/JM1
PI=14.7D0
PO=514.7D0
RB=DSQRT(RO**2-B*(RO**2-RI**2))
PHI=500.D-6
RHO=.0023D0
RPM=1800.0D0
OMEG=2.D0*PIE*RPM/60.D0
SSTIFF=23.D0*.833D0
DISP=.29125D0
SPNO=12.D0
SPLOA=SSTIFF*DISP*SPNO
ETA=2.5236D-9
TEMP=598.67D0
UCON=53.3D0*12.D0
DEM=-PIE/12.D0/ETA/UCON/TEMP

```

```

FPLOA=PIE*(PO*(RO**2-RB**2)+PI*(RB**2-RI**2))
ROOC=RO/C
PM=38000.DO
PSBAR=0.1DO*PM
RF=2.DO*(RO**3-RI**3)/3.DO/(RO**2-RI**2)
WSTAR=SPLOA+FPLOA
WRITE(6,1)
1 FORMAT(1H1,/,15X,'*****
*****',//21X,'TYPICAL INPUT PARAMETERS
*AND OUTPUT RESULTS',//,15X,'
*
WRITE(6,2) WSTAR,HA,PHI,AMAXER,WLIMIT
2 FORMAT(1X,/,15X,'REQUIRED LOAD SUPPORT =',D12.5,4X,
*'LBS',/,15X,'WAVINESS AMPLITUDE HA =',D12.5,4X,'INCH'
*,/,15X,'TILT PHI =',D12.5,17X,'IN/IN',/,15X,'PRESSURE
*CONV. CRIT. AMAXER=',F6.3,5X,'PSI',/,15X,
*'LOAD CONV. CRITER. WLIMIT =',F4.2,8X,'LBS',/)
WRITE(6,3) B,C,RPM,DT,PI,PO
3 FORMAT(15X,'BAL RATIO =',F6.3,/,15X,'HALF MAX ROUGH HEIGHT
*C =',D12.5,2X,'INCH' //15X,'RPM =',F8.1,/,15X,'HALF
*TIME STEP DT =',D12.5,8X,'SECONDS',/,15X,'PI ='
*,F8.2,7X,'PSI',/,15X,'PO =' ,F9.2,6X,'PSI',/)
RWW=RC-HA/PHI
RNO=(RWW-RI)/(RO-RI)
WRITE(6,4) RWW,RNO,RB
4 FORMAT(15X,'RADIUS OF CONTACT POINT =',D12.5,2X,'INCH'
*,/,15X,'DIMENSIONLESS OF CONTACT RADIUS =',F5.2,/,15X,
*'BALANCED RADIUS =' ,F12.5,9X,'INCH',/)
HOB=29.5D-6
CALL WLOAD(HOB)
WOB=WE
NITB=NIT
HOA=33.0D-6
CALL WLOAD(HOA)
WOA=WE
NITA=NIT
WRITE(6,5) HOA,HOB,WOA,WOB
5 FORMAT(15X,'HOA =',D12.5,2X,'INCH',16X,'HOB =',D12.5
*,2X,'INCH',/,15X,'WOA =',D12.5,2X,'LBS',17X,'WOB ='
*,D12.5,2X,'LBS',/)
100 HNEW=HOA+(HOB-HOA)*(WOA-WSTAR)/(WOA-WOB)
CALL WLOAD(HNEW)
WNEW=WE
NITN=NIT
IF(DABS(WNEW-WSTAR).LT.WLIMIT) GO TO 200
HOB=HNEW
WOB=WNEW
GO TO 100
200 WRITE(6,6) HNEW,WNEW
6 FORMAT(15X,'MINIMUM FILM THICKNESS HO=',D12.5,1X,'INCH'
*,/,15X,'FLUID LOAD SUPPORT WNEW =',D12.5,2X,'LBS',/)
WRITE(6,7) NITA,NITB,NITN
7 FORMAT(15X,'NITA =',I4,/,15X,'NITB =',I4,/,15X

```

AD-A125 810

APPLICATION OF THE WAVY MECHANICAL FACE SEAL TO  
SUBMARINE SEAL DESIGN(U) NEW MEXICO UNIV ALBUQUERQUE  
COLL OF ENGINEERING A O LEBECK ET AL. JUL 82

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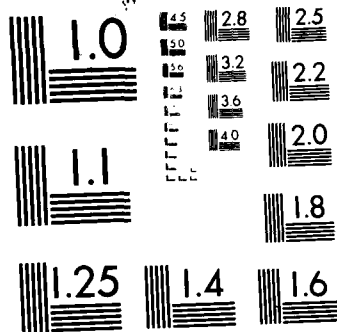
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MICROCOPY RESOLUTION TEST CHART  
NATIONAL BUREAU OF STANDARDS 1963 A

```

*, 'NITN =' , 14, //)
HST=HNEW+0.5D-6
CALL WLOAD(HST)
WOST=WE
STIFF=(WOST-WNEW)/(HST-HNEW)
WRITE(6,8) STIFF
8 FORMAT(15X, 'STIFFNESS OF FLUID =' , D12.5, 7X, 'LBS/IN' , //)
FMI=0.DO
DO 600 I=1, IM1
J=2
JMM=J-1
JP1=J+1
JP2=J+2
JP3=J+3
JP4=J+4
600 FMI=FMI+DEM*RI*H(I,1)**3*((Q(I,J)-Q(I,JMM))-(Q(I,JP1)
*-2.DO*Q(I,J)+Q(I,JMM))/2.DO+(Q(I,JP2)-3.DO*Q(I,JP1)
**3.DO*Q(I,J)-Q(I,JMM))/3.DO-(Q(I,JP3)
*-4.DO*Q(I,JP2)+6.DO*Q(I,JP1)-4.DO*Q(I,J)+Q(I,JMM))/4.DO
**+(Q(I,JP4)-5.DO*Q(I,JP3)+10.DO*Q(I,JP2)-10.DO*Q(I,JP1)
**+5.DO*Q(I,J)-Q(I,JMM))/5.DO)/DR
FMI=FMI*NN/G/RHO
WRITE(6,10) FMI
FMO=0.DO
DO 700 I=1, IM1
J=JMAX
JMM=J-1
JM2=J-2
JM3=J-3
JM4=J-4
JM5=J-5
700 FMO=FMO+DEM*RO*(H(I,JMAX)**3)*((Q(I,JMM)-Q(I,J))-(Q(I,JM2)
*-2.DO*Q(I,JMM)+Q(I,J))/2.DO+(Q(I,JM3)-3.DO*Q(I,JM2)
**3.DO*Q(I,JMM)-Q(I,J))/3.DO-(Q(I,JM4)-4.DO
**Q(I,JM3)+6.DO*Q(I,JM2)-4.DO*Q(I,JMM)+Q(I,J))/4.DO
**+(Q(I,JM5)-5.DO*Q(I,JM4)+10.DO*Q(I,JM3)-10.DO*Q(I,JM2)+
**5.DO*Q(I,JMM)-Q(I,J))/5.DO)/DR
FMO=-FMO*NN/G/RHO
FMN=FMI-FMO
WRITE(6,9) FMO, FMN
9 FORMAT(15X, 'OUTER LEAKAGE QOUT =' , D12.4, 7X, 'SCFM' , //,
*15X, 'MASS FL ERROR=' , D12.4, 13X, 'SCFM' , //)
10 FORMAT(//15X, 'INNER LEAKAGE QIN =' , D12.4, 8X, 'SCFM' , //)
CALL FRICT(PBAR, RF, AMU, FFORC, FFA, TORQ)
WRITE(6,11) AMU, FFORC, PHLS, WMECH, FFA, TORQ
11 FORMAT(15X, 'COEF OF FRICT=' , F12.8, //15X, 'FRICT FORCE=' ,
*D12.5, 26X, 'LBS' //, 15X, 'PERCENT OF FLUID LOAD =' , F10.5 //,
*15X, 'TOTAL MECH CONTACT SHEAR FORCE =' , F10.5, 9X, 'LBS' , //,
*15X, 'TOT FRICTION DUE TO ASPERITY CONTACT =' , F13.6, 'LBS' //,
*15X, 'TOTAL TORQUE =' , F13.6, 24X, 'IN-LB' , //)
DO 800 I=1, IM1
NO(I)=I
800 CONTINUE

```

```

      WRITE(6,12)
12  FORMAT(1H1,48X,'***** FLUID FILM THICKNESS *****',/60X,
      *'( INCH )',///// )
      WRITE(6,13) ((NO(IR),H(IR,JC),IR=1,IM1),JC=1,JMAX)
13  FORMAT(9(1X,I2,1X,D10.4),/)
      WRITE(6,14)
14  FORMAT(1H1,48X,'***** PRESSURE DISTRIBUTION *****'
      *,/62X,'( PSI )',///// )
      WRITE(6,13) ((NO(IR),P(IR,JC),IR=1,IM1),JC=1,JMAX)
      WRITE(6,15)
15  FORMAT(////,40X,'*****
      *****',//)
      STOP
      END

```



```

C
C
C This subroutine is used to find the pressure distribution
C   all over the seal surface.
C A transformation of tridiagonal coefficient into three
C   column matrix is also shown.
C
C
SUBROUTINE WLOAD(HMA)
  IMPLICIT REAL*8 (A-H,O-Z)
  DIMENSION NO(18),POLD(18,19)
  DIMENSION RIMH(18,19),RIPH(18,19),RJMH(18,19),RJPH(18,19)
  DIMENSION HIMH(18,19),HIPH(18,19),HJMH(18,19),HJPH(18,19)
  DIMENSION A2(18,19),A2JM1(18,19),A2JP1(18,19)
  DIMENSION A1IP1(18,19),A1(18,19),A1IM1(18,19)
  DIMENSION RJM1(18,19),RJP1(18,19),QA(18,19),PA(18,19)
  DIMENSION QIM1(18,19),QIP1(18,19),QJM1(18,19),QJP1(18,19)
  DIMENSION Z(18,5),R(18)
  DIMENSION ZZ(18,3),ZR(18)
  COMMON /BLOCKA/C,DA,DEM,DR,ETA,OMEG,PI,PO,RO,W,IMAX,JMAX
  COMMON /BLOCKB/AN,DT,HA,PHI,RC,WW,NIT
  COMMON /BLOCKC/H(18,20),P(18,20),Q(18,20),RR(18,20)
  COMMON /BLOCKD/AMAXER,RI,NN,IM1,JM1
  COMMON /BLOCKE/PFAC,PHLS,PM,TOTLD,WMECH
  IM2=IMAX-2
  JM2=JMAX-2
  IM=(IMAX+1)/2
  IMM=IM-1
  IMP=IM+1
  CON=6.DO*ETA*OMEG
  VALUE=12.DO*ETA/DT/DR
  RD=RO-RI
  PD=(PO-PI)/RD
  DO 100 J=2,JM1
  DO 100 I=1,IM1
    RJM1(I,J)=RI+(J-2)*DR
    RR(I,J)=RI+(J-1)*DR
    RIMH(I,J)=RR(I,J)
    RIPH(I,J)=RR(I,J)
    RJP1(I,J)=RI+J*DR
    RJMH(I,J)=RI+(J-1.5DO)*DR
    RJPH(I,J)=RI+(J-0.5DO)*DR
100  CONTINUE
  DO 105 J=1,JMAX
  DO 105 I=1,IM1
    RR(I,1)=RI
    RR(I,JMAX)=RO
105  CONTINUE
  DO 107 J=1,JMAX
  DO 107 I=1,IM1
    CALL WEAR(RR(I,J))
    H(I,J)=HMA+WW+(HA+(RR(I,J)-RC)*PHI)*DCOS(NN*(I-1)*DA)
107  CONTINUE

```

```

DO 109 J=2, JM1
DO 109 I=1, IM1
CALL WEAR (RR(I, J))
HIMH(I, J)=HMA+WW+(HA+(RR(I, J)-RC)*PHI)*DCOS(NN*(I-1.5DO)*DA)
HIPH(I, J)=HMA+WW+(HA+(RR(I, J)-RC)*PHI)*DCOS(NN*(I-.5DO)*DA)
CALL WEAR(RJMH(I, J))
HJMH(I, J)=HMA+WW+(HA+(RJMH(I, J)-RC)*PHI)*DCOS(NN*(I-1)*DA)
CALL WEAR(RJPH(I, J))
HJPH(I, J)=HMA+WW+(HA+(RJPH(I, J)-RC)*PHI)*DCOS(NN*(I-1)*DA)
109 CONTINUE
DO 111 KM=1, 1000
ERR=0.DO
IF(KM.NE.1) GO TO 22
DO 125 J=2, JM1
DO 125 I=1, IM1
P(I, J)=PI+PD*(J-1)*DR
POLD(I, J)=P(I, J)
Q(I, J)=P(I, J)**2
QIP1(I, J)=Q(I, J)
QIM1(I, J)=Q(I, J)
QJM1(I, J)=(PI+PD*(J-2)*DR)**2
QJP1(I, J)=(PI+PD*J*DR)**2
125 CONTINUE
DO 135 J=1, JMAX
DO 135 I=1, IM1
P(I, 1)=PI
P(I, JMAX)=PO
Q(I, 1)=PI**2
Q(I, JMAX)=PO**2
135 CONTINUE
22 CONTINUE
DO 150 J=2, JM1
NEQ=IM1
M=2
LMAX=2*M+1
DO 200 I=1, IM1
IF(I.GE.IM) GO TO 23
K=1-(1-I)*M
GO TO 21
23 IF(I.GT.IM) GO TO 24
K=IM1
GO TO 21
24 K=(IMAX-I)*M
21 AREA=RR(I, J)*DA*DR
A1(I, J)=- (HIPH(I, J)**3/RIPH(I, J)+HIMH(I, J)**3/RIMH(I, J))/DA
*-CON*(HIPH(I, J)*RIPH(I, J)-HIMH(I, J)*RIMH(I, J))/DSQRT(
*Q(I, J))-VALUE*H(I, J)*AREA/DSQRT(Q(I, J))
A1IP1(I, J)=HIPH(I, J)**3/RIPH(I, J)/DA-RIPH(I, J)*CON*HIPH
*(I, J)/DSQRT(QIP1(I, J))
A1IM1(I, J)=HIMH(I, J)**3/RIMH(I, J)/DA+CON*HIMH(I, J)*RIMH
*(I, J)/DSQRT(QIM1(I, J))
R(K)=DA /DR/DR*((RJP1(I, J)*HJPH(I, J)**3+RJMH(I, J)*
*HJMH(I, J)**3)*Q(I, J))-QJP1(I, J)*RJPH(I, J)*HJPH(I, J)**3

```

```

      *-QJM1(I,J)*RJM1(I,J)*HJM1(I,J)**3)
      *-VALUE*H(I,J)*AREA*DSQRT(Q(I,J))
      DO 210 L=1,LMAX
210  Z(K,L)=0.DO
      Z(K,M+1)=A1(I,J)
      IF(I.NE.1) GO TO 27
      Z(K,M+2)=A1IM1(I,J)
      Z(K,LMAX)=A1IP1(I,J)
      GO TO 200
27  IF(I.NE.IM1) GO TO 28
      Z(K,M)=A1IP1(I,J)
      Z(K,LMAX)=A1IM1(I,J)
      GO TO 200
28  IF(I.NE.IMM) GO TO 29
      Z(K,1)=A1IM1(I,J)
      Z(K,M+2)=A1IP1(I,J)
      GO TO 200
29  IF(I.NE.IM) GO TO 31
      Z(K,1)=A1IP1(I,J)
      Z(K,M)=A1IM1(I,J)
      GO TO 200
31  IF(I.LT.IM) GO TO 32
      Z(K,LMAX)=A1IM1(I,J)
      Z(K,1)=A1IP1(I,J)
      GO TO 200
32  Z(K,1)=A1IM1(I,J)
      Z(K,LMAX)=A1IP1(I,J)
200  CONTINUE
      CALL GAUSS(Z,R,M,NEQ)
      DO 250 I=1,IM1
      IF(I.GE.IM) GO TO 25
      K=1-(1-I)*M
      GO TO 19
25  IF(I.GT.IM) GO TO 26
      K=IM1
      GO TO 19
26  K=(IMAX-I)*M
19  Q(I,J)=R(K)
      P(I,J)=DSQRT(Q(I,J))
      IF(I.NE.1) GO TO 33
      QIP1(I,J)=R(K+M)
      QIM1(I,J)=R(K+1)
      GO TO 250
33  IF(I.LE.IM) GO TO 34
      QIM1(I,J)=R(K+M)
      IF(I.NE.IM1) GO TO 18
      QIP1(I,J)=R(K-1)
      GO TO 250
18  QIP1(I,J)=R(K-M)
      GO TO 250
34  IF(I.LT.IMM) GO TO 35
      IF(I.NE.IMM) GO TO 36
      QIP1(I,J)=R(K+1)

```

```

      QIM1(I,J)=R(K-M)
      GO TO 250
36  QIP1(I,J)=R(K-M)
      QIM1(I,J)=R(K-1)
      GO TO 250
35  QIP1(I,J)=R(K+M)
      QIM1(I,J)=R(K-M)
250 CONTINUE
150 CONTINUE
      DO 300 I=1, IM1
      M=1
      LMAX=2*M+1
      NEQ=JM2
      DO 350 J=2, JM1
      K=J-1
      AREA=RR(I,J)*DA*DR
      A2(I,J)=-DA /DR/DR*(RJPH(I,J)*HJPH(I,J)**3+RJMHI(I,J)*
*HJMH(I,J)**3)-VALUE*H(I,J)*AREA/DSQRT(Q(I,J))
      A2JP1(I,J)=DA*RJPH(I,J)*HJPH(I,J)**3 /DR/DR
      A2JMI(I,J)=DA*RJMH(I,J)*HJMH(I,J)**3 /DR/DR
      ZR(K)=Q(I,J)*((HIPH(I,J)**3/RIPH(I,J)+HIMH(I,J)**3/RIMH(
*I,J))/DA-(HIMH(I,J)*RIMH(I,J)-HIPH(I,J)*RIPH(I,J))*CON/
*DSQRT(Q(I,J)))-QIP1(I,J)*(HIPH(I,J)**3/RIPH(I,J)/DA-CON*
*HIPH(I,J)*RIPH(I,J)/DSQRT(QIP1(I,J)))-QIM1(I,J)*(HIMH(I,
*J)**3/RIMH(I,J)/DA+CON*HIMH(I,J)*RIMH(I,J)/DSQRT(QIM1(I,J)
*)))-VALUE*H(I,J)*AREA*DSQRT(Q(I,J))
      DO 310 L=1, LMAX
310  ZZ(K,L)=0.DO
      ZZ(K,M+1)=A2(I,J)
      IF(J.NE.2) GO TO 40
      ZZ(K,1)=0.DO
      ZZ(K,LMAX)=A2JP1(I,J)
      ZR(K)=ZR(K)-A2JMI(I,J)*PI*PI
      GO TO 350
40  IF(J.NE.JM1) GO TO 42
      ZZ(K,1)=A2JMI(I,J)
      ZZ(K,LMAX)=0.DO
      ZR(K)=ZR(K)-A2JP1(I,J)*PO*PO
      GO TO 350
42  ZZ(K,1)=A2JMI(I,J)
      ZZ(K,LMAX)=A2JP1(I,J)
350  CONTINUE
      CALL ZGAUSS(ZZ,ZR,M,NEQ)
      DO 400 J=2, JM1
      K=J-1
      QA(I,J)=ZR(K)
      PA(I,J)=DSQRT(QA(I,J))
      IF(J.NE.2) GO TO 43
      QJP1(I,J)=ZR(K+M)
      QJMI(I,J)=PI*PI
      GO TO 50
43  IF(J.NE.JM1) GO TO 45
      QJP1(I,J)=PO*PO

```

```

      QJM1(I,J)=ZR(K-M)
      GO TO 50
45  QJP1(I,J)=ZR(K+M)
      QJM1(I,J)=ZR(K-M)
50  DEV=DABS(PA(I,J)-POLD(I,J))
      Q(I,J)=QA(I,J)
      POLD(I,J)=PA(I,J)
      P(I,J)=PA(I,J)
      ERR=ERR+DEV
400 CONTINUE
300 CONTINUE
      IF(ERR.LT.AMAXER) GO TO 70
111 CONTINUE
      70 CALL LDTOT
      W=TOTLD
      NIT=KM
      RETURN
      END

```

```

C
C
C To find the minimum wear with respect to the same radius
C
SUBROUTINE WEAR(XR )
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION W(18)
COMMON /BLOCKA/C,DA,DEM,DR,ETA,OMEG,PI,PO,RO,WE,IMAX,JMAX
COMMON /BLOCKD/AMAXER,RI,NN,IM1,JM1
COMMON /BLOCKB/AN,DT,HA,PHI,RC,WW,NIT
DA=AN/IM1
DO 100 I=1,IM1
W(I)=(HA+(XR-RC)*PHI)*DCOS(NN*(I-1)*DA)
100 CONTINUE
SMALL=W(1)
DO 200 I=1,IM1
IF(SMALL.LE.W(I)) GO TO 200
SMALL=W(I)
200 CONTINUE
WW=-(SMALL)
RETURN
END

```

```

C
C
C To solve simultaneous equations by Gaussian Elimination
C   with tri-column matrix coefficients.
C
C
SUBROUTINE GAUSS(A,B,MBAND,NEQ)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION A(90),B(18)
CALL FSPIE
NQ1=NEQ-1
DO 10 I=1,NQ1
  I1=I+1
  I2=I+MBAND
  DO 20 II=I1,I2
    J=I
    K=(J-II+MBAND)*NEQ+II
    IF(A(K).EQ.0.DO) GO TO 20
    KK=(J-I+MBAND)*NEQ+I
    C=A(K)/A(KK)
    J1=J
    J2=J+MBAND
    IF(J2.GT.NEQ) J2=NEQ
    DO 40 JJ=J1,J2
      KKK=(JJ-II+MBAND)*NEQ+II
      KKKK=(JJ-I+MBAND)*NEQ+I
      A(KKK)=A(KKK)-C*A(KKKK)
40  CONTINUE
    B(II)=B(II)-C*B(I)
20  CONTINUE
10  CONTINUE
    K=MBAND*NEQ+NEQ
    B(NEQ)=B(NEQ)/A(K)
    DO 50 II=1,NQ1
      I=NEQ-II
      J1=I+1
      J2=I+MBAND
      S=0.0
      IF(J2.GT.NEQ) J2=NEQ
      DO 60 J=J1,J2
        K=(J-I+MBAND)*NEQ+I
60    S=S+A(K)*B(J)
        KK=MBAND*NEQ+I
50    B(I)=(B(I)-S)/A(KK)
      RETURN
    END

```

```

C
C
C To solve simultaneous equations by Gaussian Elimination
C   with tri-diagonal matrix coefficients.
C
C
SUBROUTINE ZGAUSS(A,B,MBAND,NEQ)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION A(54),B(18)
CALL FSPIE
NQ1=NEQ-1
DO 10 I=1,NQ1
  I1=I+1
  I2=I+MBAND
  DO 20 II=I1,I2
    J=I
    K=(J-II+MBAND)*NEQ+II
    IF(A(K).EQ.0.DO) GO TO 20
    KK=(J-I+MBAND)*NEQ+I
    C=A(K)/A(KK)
    J1=J
    J2=J+MBAND
    IF(J2.GT.NEQ) J2=NEQ
    DO 40 JJ=J1,J2
      KKK=(JJ-II+MBAND)*NEQ+II
      KKKK=(JJ-I+MBAND)*NEQ+I
40  A(KKK)=A(KKK)-C*A(KKKK)
      B(II)=B(II)-C*B(I)
20  CONTINUE
10  CONTINUE
    K=MBAND*NEQ+NEQ
    B(NEQ)=B(NEQ)/A(K)
    DO 50 II=1,NQ1
      I=NEQ-II
      J1=I+1
      J2=I+MBAND
      S=0.DO
      IF(J2.GT.NEQ) J2=NEQ
      DO 60 J=J1,J2
        K=(J-I+MBAND)*NEQ+I
60  S=S+A(K)*B(J)
        KK=MBAND*NEQ+I
50  B(I)=(B(I)-S)/A(KK)
    RETURN
  END

```



```

C
C
C To find the total load support including hydrodynamic,
C mechanical and shear supports, where,
C WMECH is the total mechanical load support
C TOTLD is the total load support
C PHLs is the percentage of fluid load
C W is the total fluid load support
C
C
SUBROUTINE LDTOT
IMPLICIT REAL*8 (A-H,O-Z)
COMMON /BLOCKA/C,DA,DEM,DR,ETA,OMEG,PI,PO,RO,WE,IMAX,JMAX
COMMON /BLOCKC/H(18,20),P(18,20),Q(18,20),RR(18,20)
COMMON /BLOCKD/AMAXER,RI,NWAVE,IM1,JM1
COMMON /BLOCKE/PFAC,PHLS,PM,TOTLD,WMECH
W=0.DO
WMEC=0.DO
DO 100 J=2,JMAX
DO 100 I=1,IM1
JM=J-1
HT=H(I,J)/C
CALL ASPRAT(HT,BI)
IF(I.EQ.1) GO TO 10
IF(I.EQ.IM1) GO TO 20
IIM=I-1
IIP=I+1
GO TO 30
10 IIM=IM1
IIP=I+1
GO TO 30
20 IIM=I-1
IIP=1
30 W=W+(P(IIP,J)+P(IIP,JM)+P(IIM,J)+P(IIM,JM))/4.DO*(RR(I,J)
*-DR/2.DO)*DA*DR*BI
WMEC=WMEC+PM*(1.DO-BI)*DA*DR*(RR(I,J)-DR/2.DO)
100 CONTINUE
TOTLD=(W+WMEC)*NWAVE
WMECH=WMEC*NWAVE
PHLS=W*NWAVE/TOTLD*100.DO
RETURN
END

```

```

C
C
C To find the frictional force and the coefficient of
C friction, where,
C FF is the total viscous force
C FA is the mechanical friction force
C AMU is the total coefficient of friction
C TORQ is the torque of the driving shaft
C FFORC is the total combined friction force
C
SUBROUTINE FRICT(PSBAR,RE,AMU,FFORC,FFA,TORQ)
IMPLICIT REAL*8 (A-H,O-Z)
COMMON /BLOCKA/C,DA,DEM,DR,ETA,OMEG,PI,PO,RO,WE,IMAX,JMAX
COMMON /BLOCKC/H(18,20),P(18,20),Q(18,20),RR(18,20)
COMMON /BLOCKF/FCONV,WSTAR,ROOC,NWAVE
COMMON /BLOCKG/DEL,EH,E1OH,EH3
FF=0.DO
FA=0.DO
DEL=RO*OMEG*ETA/C/PSBAR
IM1=IMAX-1
DO 100 J=2,JMAX
DO 100 I=1,IM1
R=RR(I,J)-DR/2.DO
HT=H(I,J)/C
CALL EXPT(HT)
CALL ASPRAT(HT,BI)
JM=J-1
IF(I.EQ.1) GO TO 10
IF(I.EQ.IM1) GO TO 20
IIM=I-1
IIP=I+1
GO TO 30
10 IIM=IM1
IIP=I+1
GO TO 30
20 IIM=I-1
IIP=1
30 DP=(P(IIP,J)-P(IIM,J)+P(IIP,JM)-P(IIM,JM))/4.DO/DA
FF=FF+R*R*(R*ETA*OMEG*E1OH+.5DO/R*DP*EH)*BI/RE*DA*DR
FA=FA+(1.DO-BI)*PSBAR*R*R*DA*DR/RE
100 CONTINUE
FSUM=FF+FA
AMU=FSUM/WSTAR*NWAVE
FFA=FA*NWAVE
FFORC=FSUM*NWAVE
TORQ=FSUM*RE*NWAVE
RETURN
END

```

```

C
C
C To find the expected values of film thickness
C   where E1OH is the expected value of E(1/H)
C
  SUBROUTINE EXPT(HT)
    IMPLICIT REAL*8 (A-H,O-Z)
    COMMON /BLOCKA/C,DA,DEM,DR,ETA,OMEG,PI,PO,RO,WE,IMAX,JMAX
    COMMON /BLOCKG/DEL,EH,E1OH,EH3
    EH=HT*C
    IF(EH.LT.0.DO) EH=0.DO
    EH3=HT**3*C**3
    IF(EH3.LT.0.DO) EH3=0.DO
    IF(HT.LE.1.DO) GO TO 25
    E1OH=1.DO/HT/C
    IF(HT.GT.4.DO) GO TO 50
    E1OH=(1.DO-HT*HT)**3*DLOG((HT+1.DO)/(HT-1.DO))
    E1OH=(E1OH+2.DO*HT*(33.DO+HT*HT*(15.DO*HT*HT-40.DO))
    *15.DO)*35.DO/32.DO/C
    GO TO 50
25  CONTINUE
    E1OH=-405.DO+HT*(60.DO+147.DO*HT)
    E1OH=-55.DO+HT*(132.DO+HT*(345.DO+HT*(-160.DO+HT*E1OH)))
    E1OH=(E1OH/60.DO+(1.DO-HT*HT)**3*DLOG((1.DO+HT)/DEL))*
    *35.DO/32.DO/C
50  RETURN
    END

```

```

C
C
C To find the asperity ratio of the surface where
C   BI is the fraction of seal subject to fluid pressure
C
C
SUBROUTINE ASPRAT(HT,BI)
IMPLICIT REAL*8 (A-H,O-Z)
H=HT
BI=1.DO
IF(H.GE.1.DO) GO TO 10
BI=(16.DO+H*(35.DO+H*H*(-35.DO+H*H*(21.DO-5.DO*H*H))))
*/32.DO
10 RETURN
END

```

\*\*\*\*\* The end of the whole programs \*\*\*\*\*

\*\*\*\*\*  
TYPICAL INPUT PARAMETERS AND OUTPUT RESULTS

REQUIRED LOAD SUPPORT = 0.982300 03 LBS  
 AVERAGE AMPLITUDE MA = 0.540000-05 INCH  
 TILT PHI = 0.500000-03 IN/IN  
 PRESSURE CONV. CRIT. AMAXER= 6.000 PSI  
 LOAD CONV. CRITER. WLIMIT =3.00 LBS  
  
 BAL RATIO = 0.750  
 HALF MAX ROUGH HEIGHT C = 0.200000-04 INCH  
 RPM = 1600.0  
 HALF TIME STEP DT = 0.500000-06 SECONDS  
 PI = 14.70 PSI  
 PO = 514.70 PSI  
  
 RADIUS OF CONTACT POINT = 0.192530 01 INCH  
 DIMENSIONLESS OF CONTACT RADIUS = 0.13  
 BALANCED RADIUS = 1.94857 INCH  
  
 MOA = 0.330000-04 INCH MOB = 0.295000-04 INCH  
 MOA = 0.979210 03 LBS MOB = 0.985040 03 LBS  
  
 MINIMUM FILM THICKNESS MO= 0.301760-04 INCH  
 FLUID LOAD SUPPORT WNEW = 0.982330 03 LBS  
  
 NITA = 419  
 NITB = 462  
 NITN = 454  
  
 STIFFNESS OF FLUID = 0.114040 07 LBS/IN  
  
 INNER LEAKAGE QIN = -0.12090 00 SCFM  
 OUTER LEAKAGE QOUT = -0.56600-01 SCFM  
 MASS FL ERRUK= -0.64350-01 SCFM  
  
 COEF OF FRICT= 0.00004473  
 FRICT FORCE= 0.439410-01 LBS  
 PERCENT OF FLUID LOAD = 100.00000  
 TOTAL MECH CONTACT BREAK FORCE = 0.0 LBS  
 TOT FRICTION DUE TO ASPERITY CONTACT = 0.0 LBS  
 TOTAL TORQUE = 0.087673 IN-LB

\*\*\*\*\* PARSUNG 01/10/1980 \*\*\*\*\*

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1	1	0.14700	02	2	0.14700	02	3	0.14700	02	4	0.14700	02	5	0.14700	02	6	0.14700	02	7	0.14700	02	8	0.14700	02	9	0.14700	02
10	0.14700	02	11	0.14700	02	12	0.14700	02	13	0.14700	02	14	0.14700	02	15	0.14700	02	16	0.14700	02	17	0.14700	02	18	0.14700	02	
2	1	0.23610	03	2	0.23610	03	3	0.23610	03	4	0.23610	03	5	0.23610	03	6	0.23610	03	7	0.23610	03	8	0.23610	03	9	0.23610	03
10	0.23610	03	11	0.23610	03	12	0.23610	03	13	0.23610	03	14	0.23610	03	15	0.23610	03	16	0.23610	03	17	0.23610	03	18	0.23610	03	
3	1	0.32520	03	2	0.32520	03	3	0.32520	03	4	0.32520	03	5	0.32520	03	6	0.32520	03	7	0.32520	03	8	0.32520	03	9	0.32520	03
10	0.32520	03	11	0.32520	03	12	0.32520	03	13	0.32520	03	14	0.32520	03	15	0.32520	03	16	0.32520	03	17	0.32520	03	18	0.32520	03	
4	1	0.41430	03	2	0.41430	03	3	0.41430	03	4	0.41430	03	5	0.41430	03	6	0.41430	03	7	0.41430	03	8	0.41430	03	9	0.41430	03
10	0.41430	03	11	0.41430	03	12	0.41430	03	13	0.41430	03	14	0.41430	03	15	0.41430	03	16	0.41430	03	17	0.41430	03	18	0.41430	03	
5	1	0.50340	03	2	0.50340	03	3	0.50340	03	4	0.50340	03	5	0.50340	03	6	0.50340	03	7	0.50340	03	8	0.50340	03	9	0.50340	03
10	0.50340	03	11	0.50340	03	12	0.50340	03	13	0.50340	03	14	0.50340	03	15	0.50340	03	16	0.50340	03	17	0.50340	03	18	0.50340	03	
6	1	0.59250	03	2	0.59250	03	3	0.59250	03	4	0.59250	03	5	0.59250	03	6	0.59250	03	7	0.59250	03	8	0.59250	03	9	0.59250	03
10	0.59250	03	11	0.59250	03	12	0.59250	03	13	0.59250	03	14	0.59250	03	15	0.59250	03	16	0.59250	03	17	0.59250	03	18	0.59250	03	
7	1	0.68160	03	2	0.68160	03	3	0.68160	03	4	0.68160	03	5	0.68160	03	6	0.68160	03	7	0.68160	03	8	0.68160	03	9	0.68160	03
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8	1	0.77070	03	2	0.77070	03	3	0.77070	03	4	0.77070	03	5	0.77070	03	6	0.77070	03	7	0.77070	03	8	0.77070	03	9	0.77070	03
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9	1	0.85980	03	2	0.85980	03	3	0.85980	03	4	0.85980	03	5	0.85980	03	6	0.85980	03	7	0.85980	03	8	0.85980	03	9	0.85980	03
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10	1	0.94890	03	2	0.94890	03	3	0.94890	03	4	0.94890	03	5	0.94890	03	6	0.94890	03	7	0.94890	03	8	0.94890	03	9	0.94890	03
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11	1	0.03800	03	2	0.03800	03	3	0.03800	03	4	0.03800	03	5	0.03800	03	6	0.03800	03	7	0.03800	03	8	0.03800	03	9	0.03800	03
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12	1	0.12710	03	2	0.12710	03	3	0.12710	03	4	0.12710	03	5	0.12710	03	6	0.12710	03	7	0.12710	03	8	0.12710	03	9	0.12710	03
10	0.12710	03	11	0.12710	03	12	0.12710	03	13	0.12710	03	14	0.12710	03	15	0.12710	03	16	0.12710	03	17	0.12710	03	18	0.12710	03	
13	1	0.21620	03	2	0.21620	03	3	0.21620	03	4	0.21620	03	5	0.21620	03	6	0.21620	03	7	0.21620	03	8	0.21620	03	9	0.21620	03
10	0.21620	03	11	0.21620	03	12	0.21620	03	13	0.21620	03	14	0.21620	03	15	0.21620	03	16	0.21620	03	17	0.21620	03	18	0.21620	03	
14	1	0.30530	03	2	0.30530	03	3	0.30530	03	4	0.30530	03	5	0.30530	03	6	0.30530	03	7	0.30530	03	8	0.30530	03	9	0.30530	03
10	0.30530	03	11	0.30530	03	12	0.30530	03	13	0.30530	03	14	0.30530	03	15	0.30530	03	16	0.30530	03	17	0.30530	03	18	0.30530	03	
15	1	0.39440	03	2	0.39440	03	3	0.39440	03	4	0.39440	03	5	0.39440	03	6	0.39440	03	7	0.39440	03	8	0.39440	03	9	0.39440	03
10	0.39440	03	11	0.39440	03	12	0.39440	03	13	0.39440	03	14	0.39440	03	15	0.39440	03	16	0.39440	03	17	0.39440	03	18	0.39440	03	
16	1	0.48350	03	2	0.48350	03	3	0.48350	03	4	0.48350	03	5	0.48350	03	6	0.48350	03	7	0.48350	03	8	0.48350	03	9	0.48350	03
10	0.48350	03	11	0.48350	03	12	0.48350	03	13	0.48350	03	14	0.48350	03	15	0.48350	03	16	0.48350	03	17	0.48350	03	18	0.48350	03	
17	1	0.57260	03	2	0.57260	03	3	0.57260	03	4	0.57260	03	5	0.57260	03	6	0.57260	03	7	0.57260	03	8	0.57260	03	9	0.57260	03
10	0.57260	03	11	0.57260	03	12	0.57260	03	13	0.57260	03	14	0.57260	03	15	0.57260	03	16	0.57260	03	17	0.57260	03	18	0.57260	03	
18	1	0.66170	03	2	0.66170	03	3	0.66170	03	4	0.66170	03	5	0.66170	03	6	0.66170	03	7	0.66170	03	8	0.66170	03	9	0.66170	03
10	0.66170	03	11	0.66170	03	12	0.66170	03	13	0.66170	03	14	0.66170	03	15	0.66170	03	16	0.66170	03	17	0.66170	03	18	0.66170	03	
19	1	0.75080	03	2	0.75080	03	3	0.75080	03	4	0.75080	03	5	0.75080	03	6	0.75080	03	7	0.75080	03	8	0.75080	03	9	0.75080	03
10	0.75080	03	11	0.75080	03	12	0.75080	03	13	0.75080	03	14	0.75080	03	15	0.75080	03	16	0.75080	03	17	0.75080	03	18	0.75080	03	
20	1	0.83990	03	2	0.83990	03	3	0.83990	03	4	0.83990	03	5	0.83990	03	6	0.83990	03	7	0.83990	03	8	0.83990	03	9	0.83990	03
10	0.83990	03	11	0.83990	03	12	0.83990	03	13	0.83990	03	14	0.83990	03	15	0.83990	03	16	0.83990	03	17	0.83990	03	18	0.83990	03	

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